Annual vs. Lifetime Earnings Redistribution: Does it Matter?

Fabian Kindermann
University of Regensburg,
CEPR, HCEO, and Netspar

Veronika Püschel
German Council of Economic Experts*

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Abstract

We study the economic gains and costs of progressive pensions. We compare two types of systems: The first one redistributes pension income based on an individual's annual earnings, i.e. pensions are calculated from $p = \sum_j f(y_j)$. The second one redistributes based on lifetime earnings, i.e. $p = f(\sum_j y_j)$. We evaluate these pension systems using a quantitative model that includes rich demographics such as gender, family status and children. The model provides a detailed description of individual labor supply decisions along the extensive and the intensive margin. The differences between the systems are striking. Annual earnings redistribution manages to stimulate aggregate employment, leads to less severe efficiency consequences and to a more favourable distribution of pension payments. In contrast, lifetime earnings redistribution leads to a significant reduction in employment.

JEL Classification: D15, H31, H55, J21, J22

Keywords: progressive pensions, labor supply, employment incentives

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Contact Information:
Kindermann: fabian.kindermann@ur.de, Püschel: veronika.pueschel@ur.de

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1 Introduction

Should fiscal redistribution be based on annual or lifetime earnings? This is a fundamental question in public finance and macroeconomics that has already been addressed by Vickrey (1939), who argues in favor of averaging incomes over several years for the purpose of taxation. More recently, his ideas were supported by Haan et al. (2017) or Kapička (2020), for example, who point to the economic gains of a lifetime- or a history-based tax system. Yet, while theory supports the idea of lifetime-income-based taxation, the practical implementation of such systems typically suffers from a series of obstacles, starting from a potential lack of data on the complete individual income history, over the high degree of complexity, up to legal restrictions on age discrimination.

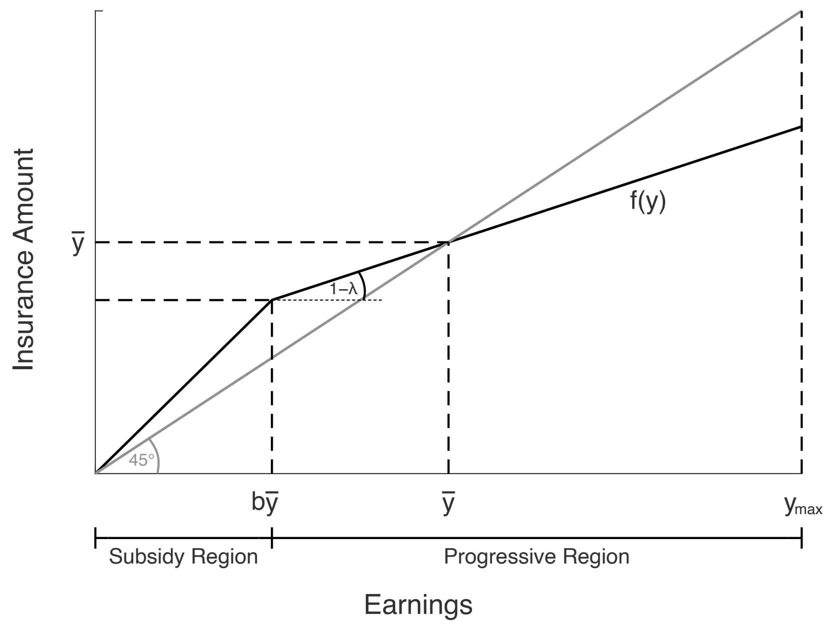
In public pension systems, however, lifetime earnings redistribution is not uncommon. Countries like the US, Portugal, or the Czech Republic support low-income earners at old age through progressive pension systems. While the details of their pension formulas differ, they all redistribute income based on information about lifetime earnings. Recently, demographic change and its consequences for public pension systems has fueled the debate about old-age poverty in several Western economies. When policy makers want to react to rising old-age poverty risk by introducing progressive pensions, they will naturally look to the examples in other countries, and above all US Social Security. But is this a good idea?

In this paper, we study the introduction of progressive pensions into an economy that, in its status quo, features a purely proportional public pension. We investigate two different types of systems: First, an *annual earnings based progressive pension* that, similar to a tax system, uses individual earnings of a given year to determine the degree of income redistribution in old age. Second, a *lifetime earnings based progressive pension* that conditions redistribution on the lifetime earnings of an individual. It turns out that the economic consequences of introducing either of the two different progressive pensions are remarkable.

Both pension systems use the same kind of progressive formula, which is inspired by the calculation formula for the primary insurance amount in US Social Security, see the black line in Figure 1. This formula takes individual earnings (either annual or lifetime average) and converts them into an insurance amount relevant for the calculation of pension claims. Relative to a proportional system (gray line), the progressive pension formula f elevates pension claims for individuals with earnings smaller than the average earnings $\bar{y}$ of the economy, and cuts them for the earnings rich. The pension of the average earner remains untouched. In the low earnings segment, the formula features a subsidy region, in which workers accumulate disproportionately high pension claims up to a threshold level $b\bar{y}$. Beyond this threshold, additional earnings only translate into pensions claims with a factor $1 - \lambda$, where $0 \leq \lambda \leq 1$ is a measure for the degree of pension progressivity.[1]

[1] A value of $\lambda = 0$ resembles a proportional pension, whereas under $\lambda = 1$ the pension system

Figure 1: Progressive pension formula



We quantify the economic consequences of introducing either annual or lifetime earnings based progressive pensions into the German economy. Germany is the biggest economy in Europe and awaits a demographic transition that is representative of many Western economies. It currently runs a public pay-as-you-go pension system with a proportional pension formula. Yet, owing to the pension reforms of the early 2000s, which were made to deal with future demographic pressure, old-age poverty rates are projected to increase, see Haan et al. (2017). This in turn fuels a debate about how the German pension system can be adjusted in order to deal with increased old-age poverty risk, see for example BMWi (2021). A progressive pension system certainly is one way to deal with this problem.

We evaluate our proposed pension reforms using a quantitative macroeconomic model with heterogeneous agents and overlapping generations. Our model focuses on two dimensions that we consider to be of first-order importance when studying progressive pension reforms: First, in order to provide an adequate picture of the heterogeneity of pensioners and their risk of poverty in old age, we account for a wide range of demographic and labor market characteristics. We consider individuals of different genders who can either live as single households or in a marriage. Both single and married women may give birth to children and raising children is costly in terms of time and resources. We distinguish individuals according to their education and account for assortative mating in the marriage. Furthermore, we allow for differences in labor productivity that arise from persistent

would be fully flat beyond the threshold level $b\bar{y}$.

productivity shocks, gender discrimination and a motherhood wage penalty. The second important building block of our quantitative analysis is a detailed model of individual labor supply decisions. We model labor supply decisions along the extensive margin and for women we also look at the intensive margin. Couple households have to solve a two-earner problem, where we allow for specialization especially under the presence of young children. Women can choose to work on full-time, part-time or marginal working contracts. Not working on a full-time job may have intertemporal spill-overs in the sense that a woman's future option to work full time may be restricted (at least for a while). Using this detailed framework, our simulation model matches empirical labor supply profiles of different population subgroups, the motherhood penalty as well as the distribution of pension claims of the German economy.

Our simulation results reveal significant differences between a progressive pension in which redistribution is based on annual earnings and one in which redistribution is based on lifetime earnings. The most important difference relates to the labor supply incentives embedded in both systems. In comparison to a proportional pension, both progressive pension systems distort labor supply along the *intensive margin*, meaning that women generally reduce their working hours. This effect is standard and relates to the equity-efficiency trade-off embedded in all progressive fiscal systems. It results from the fact that the link between labor earnings and pension claims is weakened in the progressive region of the pension formula as outlined in Figure 1. Along the *extensive margin*, however, the two systems come with entirely different incentives. Under an annual earnings based progressive pension, an individual has to provide positive labor hours in order to make it into the subsidy region. The disproportionately high accumulation of pensions claims in this subsidy region, hence, stimulates labor force participation of individuals that are on the margin of dropping-out of the labor force. Those are predominantly mothers who are taking care of children and elderly workers who would like to retire early either because of a negative productivity shock or because they just were very lucky in their earlier working years. In the aggregate, this stimulative subsidy effect causes an increase in aggregate employment. On the other hand, whether an individual ends up in the subsidy region or the progressive region under lifetime earnings redistribution is not determined by the worker's current labor market situation, but her or his (expected) lifetime earnings. Hence, when on the margin of being employed or not, workers who already had a good track record or who expect the situation to improve in the future are likely to drop out of the labor force (either temporarily or permanently). In doing so, they are (partly) compensated for the missing years of contributions through the progressive pension formula. As a result, aggregate employment falls.

The employment effects embedded in these two progressive pension systems turn out to be of first-order relevance along several dimensions. First, the stimulation of aggregate employment under an annual earnings based progressive pension limits the negative macroeconomic efficiency consequences of increased redistribution. Consequently, long-run macroeconomic outcomes are much better under annual earnings redistribution than under lifetime earnings redistribution. Second, the stimulative employment effect also has consequences for the distribution of pension claims. While the introduction of progressive pensions generally leads to a smaller amount of old-age income inequality, the annual earnings based progressive pension creates a superior distribution of pension claims with a smaller left and a fatter right tail. It stimulates employment especially for workers who would otherwise only work in marginal employment or not at all. The additional working years paired with a sizeable earnings subsidy successfully reduces the very left tail of the old-age income distribution. Under lifetime earnings redistribution, the very same workers rather drop out of the labor force (either temporarily or permanently) as they are (partly) compensated for missing years of contributions through the progressive pension formula. Yet, this compensation does not make up for the entire loss in pension entitlements from working a year less, which in turn leads to more mass on the left tail of the pension distribution. Summing up, a lifetime earnings based system would have the potential to create a more even distribution of pension payments by conditioning redistribution on permanent rather than transitory income. It fails to do so, however, because of its negative employment incentives on low productivity workers. In the end, keeping up work turns out to be a better insurance against the risk of old-age poverty than government redistribution. It is hence not surprising that a pension system with annual income redistribution leads to long-run welfare gains, while lifetime earnings redistribution causes welfare losses. The difference in welfare effects between the two systems amounts to almost 1 percent measured in consumption equivalent variation.

Relation to the literature We add to a literature that examines the welfare consequences of changing the redistributive properties of social security systems in heterogeneous-agent life cycle models, among them Huggett and Ventura (1999), Fehr et al. (2013), Nishiyama and Smetters (2008), O'Dea (2018), or Jones and Li (2022). Relative to these studies, we are among the first to study extensive margin effects of social security reforms. The only other study we know so far is Gustafsson (2023) who examines the introduction of a purely Beveridgean pension system. Our paper more generally connects to the literature on extensive margin labor supply responses and the role for the fiscal tax and redistribution system. Saez (2002) was among the first to show that, when labor supply responses are concentrated along the extensive margin, an optimal labor tax policy explicitly subsidizes employment in a similar way as the Earned Income Tax Credit in the US. A series of studies has quantified the EITC's impact on labor supply, savings, insurance and welfare, including Chan (2013), Athreya et al. (2010), and Ortigueira and Siassi (2022).

The empirical literature has validated that pension reforms can trigger individual labor supply reactions, see for example Coile (2015), Blundell et al. (2016) and Liebman et al. (2009). A recent study by French et al. (2021) exploits a 1999 pension reform in Poland and confirms that labor supply incentives embedded in pension reforms trigger behavioral recations even about 15 years prior to retirement entry.

Finally, our paper relates to a recent literature that uses large scale quantitative simulation models with very detailed heterogeneity on the household level. These studies analyze the impact of public policies on individuals of different gender or family type. Examples include Guner et al. (2021), Kaygusuz (2015) or Kurnaz (2021). Amongst them, Kaygusuz (2015) is the closest to our study, as he investigates changes in redistributive features of US Social security. However, he abstracts from any sort of earnings risk and can hence not quantify insurance effect

The remainder of our paper is structured as follows: In Section 2, we analytically investigate the incentive effects embedded in the different progressive pension systems. In Section 3, we present our full quantitative simulation model and its calibration. Section 4 discusses simulation results and the last section concludes.

2 Building Intuition

Before setting out our large-scale simulation model, we want to build some intuition. To this end, we study a three-period stylized model in which we illustrate the main mechanisms that will shape life-cycle labor supply under the different progressive pension systems discussed in the introduction. Households in this framework live for three periods $j = 1, 2, 3$. They supply labor and contribute to the pension system in the first two periods of life, in the final period they are retired. The wage rate w_j for effective labor is exogenous. Individuals derive utility from consumption c_j in each period and suffer disutility from working ℓ_j. For analytical tractability, we assume that preferences are quasi-linear in consumption and that the time discount rate equals the interest rate $r = 1$.[2] Disutility from labor is governed by the Frisch elasticity of labor supply χ. More specifically, we let preferences be represented by the utility function

$$U(c_1, c_2, c_3, \ell_1, \ell_2) = c_1 + c_2 + c_3 - \frac{\ell_1^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \frac{\ell_2^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}. \tag{1}$$

Households maximize utility in (1) subject to the present value budget constraint

$$c_1 + c_2 + c_3 = (1 - \tau_p)(w_1\ell_1 + w_2\ell_2) + p. \tag{2}$$

The mechanics of the pension system in this setup are quite simple. The system collects contributions in the form of a payroll tax τ_p on earnings $w_j\ell_j$ from workers. In reward for their annual contributions, workers are credited a pension based on their earnings history.

[2] We relax all of these assumptions later on in our quantitative model.

2.1 Lifetime Earnings Based Progressive Pensions

In a lifetime earnings based progressive pension, the progressive pension formula $f(\cdot)$ from Figure 1 is applied to average lifetime earnings $y = \frac{w_1\ell_1+w_2\ell_2}{2}$. In analytical terms, this formula reads

$$f(y) = \begin{cases} \left[\frac{\lambda}{b} + (1-\lambda)\right]y & \text{in the subsidy region } y < b\bar{y} \text{ and} \\ \lambda\bar{y} + (1-\lambda)y & \text{in the progressive region.} \end{cases} \quad (3)$$

The individual pension payment can then be calculated from

$$p = \kappa \times f\left(\frac{w_1\ell_1 + w_2\ell_2}{2}\right),$$

where κ denotes the pension system's replacement rate. Plugging this into the household's budget constraint, we can calculate the individual labor supply choice as[3]

$$\ell_j = \begin{cases} \left[w_j \times \left(1 + \underbrace{\tau_p\left(\frac{1}{b} - 1\right)\lambda}_{\tau_p^{sub}}\right)\right]^{\chi} & \text{if } \frac{w_1\ell_1+w_2\ell_2}{2} < b\bar{y} \text{ and} \\ \left[w_j \times \left(1 - \underbrace{\tau_p\lambda}_{\tau_p^{prog}}\right)\right]^{\chi} & \text{otherwise.} \end{cases} \quad (4)$$

The two regions of the progressive pension formula are mirrored in the policy function for labor supply. Individuals in the subsidy region experience a *subsidy to working* $\tau_p^{sub} = \tau_p\left(\frac{1}{b} - 1\right)\lambda \geq 0$, which encourages labor supply. Individuals in the progressive region face an implicit tax $\tau_p^{prog} = \tau_p\lambda \geq 0$ that distorts labor supply downwards. The extent of the labor supply distortion τ_p^{prog} crucially depends on the degree of progressivity λ. A higher pension progressivity flattens the link between individual earnings and pension payments. Consequently, a larger part of pension contributions is perceived as a tax by the individual. The labor subsidy τ_p^{sub} in addition varies with the bend point b of the progressive pension formula. The smaller is b the steeper is the slope in the subsidy region. Hence, positive labor supply incentives within the subsidy region expand, while the overall size of the subsidy region shrinks.

Crucially, whether an individual is located in the subsidy or the progressive region solely depends on the individual's lifetime average earnings. This means that only individuals with permanently low earnings can benefit from the pension subsidy. Individuals with high permanent earnings that face a temporary low earnings episode, on the other hand, remain located in the progressive region and can't experience a positive incentive effect. Hence, labor supply in a given period is not only determined by the current wage, but also by earnings in all other periods.

[3]Note that we deliberately assumed that $\frac{\kappa}{2\tau_p} = 1$, meaning that the average return on a pension is the same as that of the capital market. This is purely for illustrative purposes and this restriction will be relaxed in our quantitative model.

2.2 Annual Earnings Based Progressive Pensions

In an annual earnings based progressive pension system, the progressive pension formula $f(\cdot)$ is applied to annual earnings $y_j = w_j l_j$. Consequently, the individual pension payment reads

$$p = \kappa \times \frac{f(w_1\ell_1) + f(w_2\ell_2)}{2}.$$

With this pension formula, the individual labor supply decision is

$$\ell_j = \begin{cases} \left[w_j \times \left(1 + \underbrace{\tau_p\left(\frac{1}{b} - 1\right)\lambda}_{\tau_p^{sub}}\right)\right]^\chi & \text{if } w_j\ell_j < b\bar{y} \text{ and} \\ \left[w_j \times \left(1 - \underbrace{\tau_p\lambda}_{\tau_p^{prog}}\right)\right]^\chi & \text{otherwise.} \end{cases} \tag{5}$$

While the labor supply policy function looks almost like the one in (4), there is one important difference: whether an individual is located in the subsidy or the progressive region depends solely on instantaneous earnings and not on lifetime average earnings. Hence, workers that experience a temporary low earnings episode can enjoy the positive incentive effects of the subsidy region, too. This provides additional incentives to increase labor supply for all individuals with low earnings. In turn, the labor supply decision in a given period j is only determined by the current wage w_j and is not restricted by what happens in other periods along the life cycle.

The annual earnings based progressive pension is very closely related to the Earned-Income Tax Credit (EITC) in the US. The functional form that features a phase-in and phase-out structure, the assessment base (annual earnings) and the labor supply incentives (both at intensive and also the extensive margin) can be expected to be similar. The main difference is that the benefits are distributed during retirement rather than working life. Hence, we can also call the latter system an Earned-Income Pension Credit (EIPC).

2.3 Annual vs. Lifetime Earnings Redistribution

Should old-age income redistribution be based on annual or on lifetime average earnings earnings? The previous theoretical considerations suggest that an annual redistribution base leads to more favourable labor market outcomes. While the incentive effects of both types of progressive systems are similar for individuals with permanently low earnings, they differ for individuals with temporary low earnings episodes. In a lifetime earnings based system, a worker with a one-time adverse productivity shock is treated the same as in any other period. If her lifetime average earnings are large enough, a progressive pension system still imposes a negative tax on her labor supply, and she might therefore choose to work less or not at all. A system based on annual earnings provides a subsidy to labor supply of all workers with low earnings, regardless of their lifetime income. Yet, to become eligible for the subsidy, an individual must work especially during times of low earnings.

While its labor market outcomes can be expected to be less great, lifetime based redistribution should also provide benefits as it allows for a better targeting towards workers permanently in need of funds. As such, a lifetime based system allows for the separation of redistribution and insurance concerns, which are inherently mingled when annual income is used as a redistribution base. This creates a trade-off between labor supply incentives and redistribution goals.

In the remainder part of this paper, we will study both the incentive and redistributive aspects of progressive pension systems in which the distribution is based on either lifetime or annual earnings. To paint a credible picture of both the incentive and redistribution concerns, we use a simulation model with a lot of demographic detail. We consider men and women, single and couple households, and individuals with and without children. Doing so allows us to point to the parts of the population most in need of resources at old age as well as those groups most prone to labor market incentives.

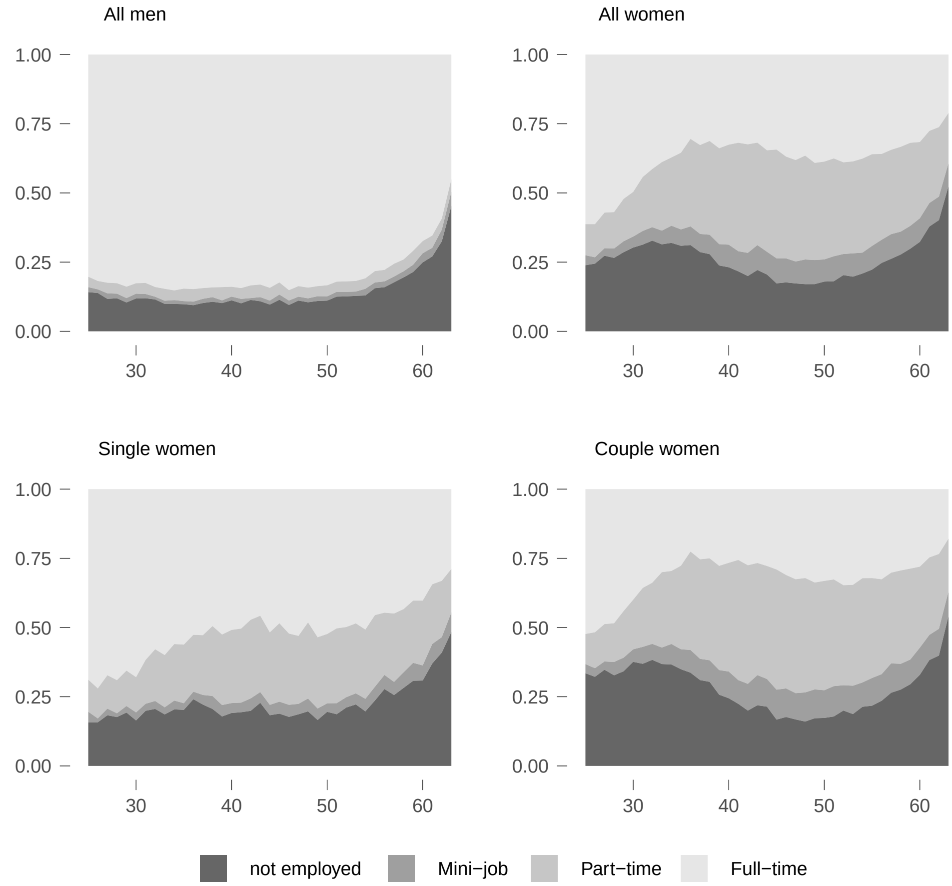
3 The Quantitative Simulation Model

Our quantitative analysis is motivated by the fact that empirical life-cycle labor supply patterns are quite distinct across demographic groups. Figure 2 shows data from the 2017 German Microcensus[4] on the distribution of labor hours and job characteristics of men as well as single and married women. There are some stark differences that immediately stand out. For men, non-employment continuously rises with age – especially so as individuals approach retirement – and part-time work is virtually non-existent. Non-employment for women, on the other hand, exhibits a wave-shape over the life cycle, which is predominantly driven by married mothers taking care of young children in their late 20s and 30s. Part-time work and mini-jobs – a special form of tax-favored, low-hours marginal employment – are much more prevalent for women than for men. As one would expect, the data is consistent with the notion that the arrival and presence of young children goes along with a decline in full-time work. Interestingly however, the data also suggest that women hardly return to full-time work as children become older. Thus, only about 50 percent of single women and 25 percent of married women work on a full-time contract starting from age 40 up until retirement.

Our full quantitative simulation model intents to speak to these empirical observations. It is populated by overlapping generations of households that can be distinguished according to a rich set of demographics. At the beginning of their (economically active) life, individuals are endowed with a gender and an education

[4]See RDC of the Federal Statistical Office and Statistical Offices of the Federal States (2021)

Figure 2: Empirical labor supply



level. Based on these characteristics, they may get married to a partner or stay single. Throughout their life cycle, individuals are exposed to idiosyncratic labor productivity shocks. In addition, based on their partnership status and education, they may give birth to children. Children cause both time and monetary costs to parents. Throughout their life, and especially so during retirement, households are exposed to survival risk. We assume that married partners die together.

Households decide about how much to work, consume and save. Decisions in a couple are made jointly, and partners can choose how to allocate labor hours across individuals. The labor supply decision features both an extensive and an intensive margin choice. Staying at home or working part time today may have intertemporal spill-overs in the sense that an individual's option to work full time in any future period may be restricted.

The government operates a pay-as-you-go pension system financed by payroll taxes. In addition, it collects revenue through the progressive taxation of labor earnings and a proportional consumption tax to cover general government expenditure and transfer payments to families with kids. We consider an open economy framework, so that the prices for capital and labor are fixed, but government parameters adjust in order to keep the fiscal tax and transfer systems balanced.

In the following, we provide a detailed description of model assumptions and equations. In addition, we immediately discuss parameter choices. Owing to the richness of the model, we will remain very brief on the choices of parameters that can be considered as "standard" in the quantitative life-cycle model literature. Yet, Appendix B provides a detailed and thorough description of parameter choices as well as their empirical targets. Our base year is 2017, in which the average contributory earnings – the empirical counterpart to average earnings of the employed in our model – amounted to EUR 37,000, see DRV Bund (2020). Since we only consider long-run equilibria, we omit the time index t in the following wherever possible.

3.1 Demographics

Age, gender, education and marriage The economy is populated by overlapping generations of heterogeneous individuals, like in Auerbach and Kotlikoff (1987). At each point in time t, a new generation is born. We normalize cohort sizes to 1. Individuals start their economic life at age $j = 20$ and we allow for a maximum life span of $J = 99$ years. They enter the economy as either male or female $g \in \{m, f\}$ with either high-school or college education $s \in \{0, 1\}$. Based on these characteristics, they are potentially matched with a partner of the opposite gender to form a married couple. Marriages are stable over the life cycle and couples die jointly. If not matched to a partner, individuals form a single household. We denote the household type (single or couple) by $i \in \{s, c\}$. Gender, education and marital status constitute the permanent household characteristics.

We use data from the age cohorts 35-49 of the 2017 German Microcensus to estimate the following demographic parameters:

1. 50.78% of individuals in the sample are male, 33.02% of them have a college education, and 67.70% of them live in a couple household.[5]
2. The proportion of women is 49.22% and 27.70% of them have a college education.
3. We find a considerably degree of assortative mating, meaning that 85.69% of non-college educated men are married to a non-college educated woman and 54.81% of college-educated men are married to a college-educated woman.

We choose our model parameters to be consistent with these observations, see Appendix B.1.1.

Retirement and survival Individuals can supply labor to the market until they reach the mandatory retirement age $j_r = 64$, which corresponds to the av-

[5] This includes individuals who live in a couple household but are not formally married.

erage retirement age of the German regular retirement population in 2017, see DRV Bund (2019). Whether a household is still alive in the next period is uncertain and depends on survival probabilities that are specific to age, gender and marital status. We denote by $\psi^i_{j,g}$ the conditional probability of a household to survive from period $j-1$ to period j, with $\psi^i_{20,g} = 1$ and $\psi^i_{J+1,g} = 0$. We directly extract survival probabilities of singles from the 2017 annual life tables of the Human Mortality Database (2020) and use the average survival probability across genders for couples, see Appendix B.1.2 for details. Owing to the death of households, cohort sizes shrink with age. We therefore let m_j be the relative size of the cohort aged j.

The arrival and presence of children At each working age j, a women that did not have kids before may give birth to two children. Fertility and child-rearing are modeled as a stochastic process. In particular, we assume the state k to take four distinct values. $k = 0$ indicates non-mothers, who might have kids in any future period. A mother with young children (0-5 years) has the state $k = 1$. When kids become older (aged 6-17), mothers stochastically transition into $k = 2$. Finally, $k = 3$ indicates mothers whose children have already left the household. The transition probabilities $\pi_k(k^+|k, j, i)$ govern the child-bearing and -rearing process. They depend on age j and marital status i. Note that $k = 3$ is an absorbing state. Kids never enter the economy as productive agents.

According to the 2017 German Microcensus, 80.02% of married women and 47.53% of single women in the cohort 35-49 had at least one child present in their household. We use these numbers as proxies for the overall likelihood of giving birth to children. Furthermore, we extracted data from Eurostat (2023) on mothers' age at first birth, which we use to infer age-specific child-birth probabilities. Together these data lead us to life-cycle child-birth probabilities $\phi^k_{i,j}$ that cause a transition from state $k = 0$ to state $k = 1$, see Appendix B.1.3 for details.

3.2 The Structure of the Labor Market

Labor hours choices Every individual has a time endowment equal to 1. Men can either choose to work full time (ℓ_{full}) or not at all.[6] Women, on the other hand, can choose from a menu of working hours $\{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$. ℓ_{part} corresponds to part-time work and ℓ_{mini} represents a mini job, a special form of marginal employment under which workers are allowed to earn 450 Euros tax free. According to the German Microcensus, full-time employees on average work for a fraction $\ell_{full} = 0.404$ of their total time endowment, whereas part-time employees work $\ell_{part} = 0.213$, see Appendix B.2.1. For mini jobs, we set $\ell_{mini} = 0.1$ paying tribute to the fact that those jobs are typically low-hours marginal types of employment.

[6] In addition to computational ease, this choice is grounded in the empirically negligible share of men working part time, see Figure 2.

Labor market flexibility If a woman chooses not to work full time, this may restrict her future hours choices. We denote by h a woman's state of labor market flexibility at a given age. If $h = \ell_{full}$, then she can choose from the entire set of available hours. If $h = \ell_{part}$, then she is restricted in her choices and can only work part time, marginally or not at all. By modeling restricted working hours, we pay tribute to the fact that many women – and especially mothers – continue to work part time at older ages, even when their children have already left the household, see for example Gallego Granados et al. (2019). We model the transition of h over ages as a first-order discrete Markov process with transition probabilities $\pi_h(h^+|h, g, \ell)$. For women who work full time, we assume that the transition matrix is the identity matrix, meaning that they will not be at the risk of facing labor hours restrictions in the next period. Women who do not work full-time transition from $h = \ell_{full}$ into the state $h = \ell_{part}$ with a likelihood of 0.95. Once in this state, they come back to $h = \ell_{full}$ with an annual probability of 0.15. The average duration of a period of labor hours inflexibility is therefore 6.67 years, meaning that $h = \ell_{part}$ is not an absorbing state for a woman. Instead, we can think of it as her not being able to find a full-time work position quickly or her employer pushing her to remain part time. Hence, the labor market restriction may resolve over time. Our choice of parameters for the transition process between states ensures that the fraction of women working part-time or less, even when their children have already left the home, coincides with the empirical observations in Figure 2.

General labor productivity Households are ex ante homogeneous, but differ ex post in their labor productivity. All individuals of a given education level s share a common deterministic age-specific labor productivity profile $\theta_{j,s}$. Throughout working life, they are subject to idiosyncratic productivity shocks η, which follow a standard AR(1) process in logs

$$\eta^+ = \rho_s \eta + \varepsilon^+ \quad \text{with} \quad \varepsilon^+ \sim N(0, \sigma_{\varepsilon^2,s}), \tag{6}$$

where innovations ε^+ are iid across and within households. $\pi_\eta(\eta^+|\eta, s)$ denotes the probability distribution of next-period's productivity η^+, conditional on current labor productivity η and education s. We denote by $z(j, s, \eta) = \exp(\theta_{j,s} + \eta)$ the general productivity level of an individual at age j, education s and labor productivity shock η. A man's wage is then simply the product of the marginal product of labor w and general productivity, i.e.

$$w(j, s, \eta, m, 0) = w \times z(j, s, \eta).$$

We estimate the life-cycle labor productivity profiles $\theta_{j,s}$ as well as the idisyncratic productivity risk process η from administrative data – the scientific use file of the Versichertenkontenstichprobe 2017 (FDZ-RV, 2017) – on male earners from the German public pension insurance (Deutsche Rentenversicherung). On these data, we first run a regression with time and age fixed-effects to recover the average life-cycle labor earnings profile. We then use the residuals to parameterize the stochastic component of labor productivity. The left panel of Figure 3 shows the empirical and the model simulated life-cycle labor earnings profiles for men. Table 1 summarizes the parameters for labor productivity risk. As usually found in the literature, the processes for labor productivity risk are highly persistent, with a somewhat smaller persistence for high-school workers and a larger persistence for college graduates. The overall unconditional process variance ranges at around 28 to 30 log-points. Appendix B.2.2 provides more details on the estimation procedure.

Figure 3: Empirical and model implied average life cycle earnings profiles

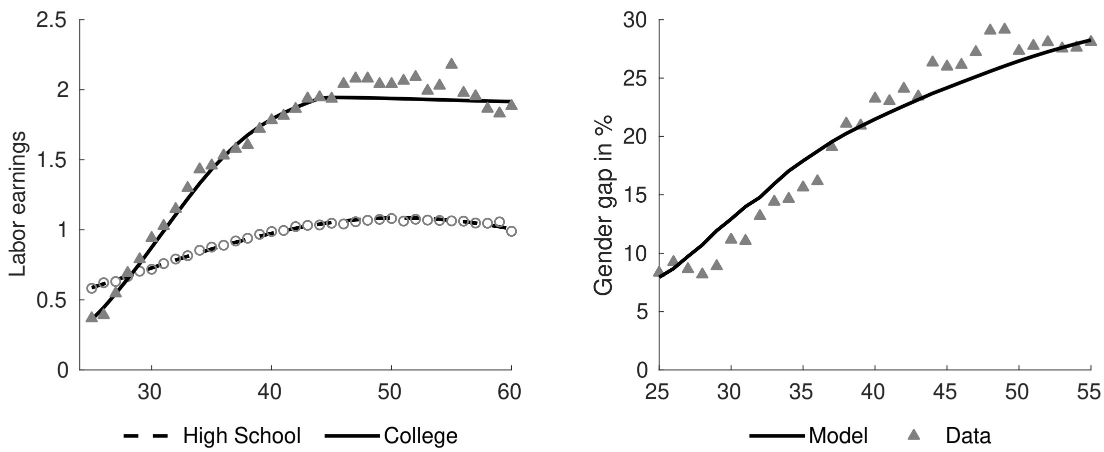


Table 1: Parameter values of labor productivity risk

	High School $s = 0$	College $s = 1$
Autocorrelation ρ_s	0.9300	0.9900
Innovation Variance $\sigma^2_{\varepsilon,s}$	0.0372	0.0059
Unconditional Variance $\frac{\hat{\sigma}^2_{\varepsilon,s}}{1-\hat{\rho}^2_s}$	0.2754	0.2965

Gender wage gap Women face a lower wage level owing to a general gender wage gap w_{gap}. In addition, following evidence in the empirical literature, see for example Kleven et al. (2019), we assume that there is a motherhood wage penalty $w_{pen}(k)$ that depends on a woman's child-bearing and -rearing state k. Consequently, the wage a woman earns at any given state is given by

$$w(j, s, \eta, f, k) = w \times w_{gap} \times w_{pen}(k) \times z(j, s, \eta).$$

Note that by using this formulation, we assume, strictly speaking, that a woman with the same age-education-productivity-shock combination as a man is less productive. In reality, of course, there are several explanation for varying wage levels between men and women as well as mothers and non-mothers. This includes taste-based discrimination, less opportunities for human capital accumulation during child-rearing years, career ladder aspects or selection into different jobs based on individual fertility preferences as in Adda et al. (2017). Including all these channels, however, would go well beyond the scope of this paper.

The gray triangles in the right panel of Figure 3 show the gross wage gap between employed men and women over the life cycle as estimated by Schrenker and Zucco (2020). To account for these age-specific wage differences, we calibrate $w_{gap} = 0.87$ as well as a motherhood wage penalty that depends on the presence and age of children as

$$w_{pen}(\cdot) = \begin{bmatrix} 1.00 & 0.91 & 0.75 & 0.62 \end{bmatrix}.$$

Note that the *motherhood penalty* estimated in empirical studies typically combines the effects of hours and wage changes into one statistic. In contrast, our estimates of the pure *motherhood wage penalty* solely focus on wage differentials between mothers and non-mothers, which rather are a consequence of missed opportunities for accumulating specific human capital or climbing the career ladder. As such, the motherhood wage penalty rises with the age of children and is the largest when children have left the house. Using these estimates, our model provides a good fit for the evolution of the gender wage gap over the life cycle, see the right panel of Figure 3. In Section 4.2 we also provide model simulations for the entire motherhood penalty that combines wage and hours differentials.

Labor earnings Labor earnings of an individual are finally calculated from this person's wage and his or her working hours

$$y_{j,g} = w(j, s, \eta, g, k) \times \ell_{j,g}.$$

In addition to regular working contracts, German tax and social security law allows for a special type of marginal employment, so-called mini jobs. These jobs typically feature a small number of working hours, and workers can not earn more than a certain amount $\bar{y}_{mini}$. In turn, such jobs are exempt from income taxation and subject to reduced social contributions. For workers in this marginal employment category we calculate labor earnings as

$$y_{mini,j,g} = \min\left[w(j, s, \eta, g, k) \times \ell_{mini}\ ,\ \bar{y}_{mini}\right]$$

and set their regular earnings to zero. Note that this formulation means that high productivity individuals can not work at their full labor productivity in such jobs. According to German tax law, there is a earnings-threshold for mini jobs of 5400 Euros annually. Since not every mini-job worker earns the maximum amount, we assume mini-job earnings of 400 Euros per month that corresponds to 4800 Euros annually or $\bar{y}_{mini} = 0.1297 \times \bar{y}$. $\bar{y}$ denotes average earnings of the employed.

3.3 Preferences and the Budget Constraint

Preferences Individuals have preferences over stochastic streams of consumption $c_{j,g} \geq 0$ and labor supply $\ell_{j,g} \geq 0$. Single households maximize the discounted expected utility

$$U_0^s = E_0 \left[\sum_{j=20}^{J} \psi_{j+1,g}^s \beta^{j-19} u(c_{j,g}, \ell_{j,g}, \xi) \right],$$

and couple households maximize the sum of individual discounted utilities, where we assumed that partners always die jointly. Expectations are formed with respect to future labor productivity, the future labor market flexibility of women, labor force participation costs, as well as the presence of children. Households discount the future with the constant time discount factor β as well as their individual survival rate.

We assume a period utility function

$$u(c_{j,g}, \ell_{j,g}, \xi) = \frac{c_{j,g}^{1-\sigma}}{1-\sigma} - \nu_g \frac{(\zeta_{k,i,g} + \ell_{j,g})^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell_{j,g}>0}. \tag{7}$$

Utility is additively separable in consumption $c_{j,g}$ and labor supply $\ell_{j,g}$. Utility from consumption features constant absolute risk aversion σ, utility from labor a constant but gender-specific Frisch elasticity χ_g. Participation in the labor market is costly to individuals. Specifically, when choosing labor hours greater than zero, a worker has to pay the participation utility cost ξ. We assume that ξ is drawn at the household-level – meaning that it is common to married couples – but iid across households and across time and independent of individual labor productivity. We let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2.

We assign a value of 2 to risk aversion σ, a choice quite typical for the heterogeneous agent macroeconomics literature though at the lower end of values that generate an extensive desire for redistribution.[7] The empirical literature has pointed to the fact that Frisch elasticities differ significantly between men and women, see for example Keane (2011). Consistent with this evidence, we chose values of $\chi_m = 0.4$ for men and $\chi_f = 0.75$ for women. After making these data-based choices, we are left with the set $(\beta, \nu_m, \nu_f, \mu_\xi, \sigma_\xi^2)$ of parameters that we need to calibrate. We choose the time discount factor $\beta = 0.9785$ so that all capital is entirely absorbed by private savings in the initial equilibrium. We then jointly calibrate $\nu_m = 70$, $\mu_\xi = 1.65$ and $\sigma_\xi^2 = 2.5$ to match the participation rates of men across demographic groups in Table 7 in Appendix B.3. Finally we

[7] In this model, σ fulfils two roles as it defines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter typically range between values of 1 and 3, whereas risk aversion can be quite high and well beyond values of 10 when estimated from individual financial choices, see for example Vissing-Jørgensen and Attanasio (2003).

set $\nu_f = 22.0$ to achieve an overall good divide between mini-job, part-time and full-time work for women.

Families and Children Families enjoy economies of scale in consumption.[8] Aggregate household consumption expenditure can be calculated from

$$c = c_g \times \upsilon(j, k, i),$$

where $\upsilon(j, k, i)$ is a scale factor that depends on the age and the composition of the household. We apply the new OECD equivalence scale.[9] This means that in order to realize the same individual consumption level, larger families face a smaller per capita spending. Children need to be fed. Consequently, they exhibit consumption costs through the scale factor $\upsilon(j, k, i)$.[10]

In addition to being fed, children also need to be looked after. Hence, the presence of children comes with time costs $\zeta_{k,i,g}$ that depend on the children's age. Time costs are fully borne by single mothers, but they can be partly shared by married couples. We calibrate the time cost of young children $\zeta_{1,s,f} = 1.50$ and older children $\zeta_{2,s,f} = 0.25$ to match the labor supply patterns of both single women as well as young women in Table 7 in Appendix B.3. In order to match the empirical labor supply profiles of married women, too, we assume that the overall time costs of raising children are the same in single and married couple families, but that fathers take a certain (small) share of these costs. This leads us to $\zeta_{1,c,f} = 1.20$ and $\zeta_{1,c,m} = 0.30$ for young children as well as $\zeta_{2,c,f} = 0.1875$ and $\zeta_{2,c,m} = 0.0625$ for older children.

Budget constraint Markets are incomplete. Like in Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994), households can only self-insure against fluctuations in individual states by saving in a risk-free asset a with return r. They cannot borrow, so that assets must satisfy $a \geq 0$. A household's resources are composed of current wealth a (including returns), income from working in regular jobs y or in marginal employment y_{mini}, pension payments p, government transfers $t(k, i)$, and intergenerational transfers b.[11] They use these resources to finance consumption expenditure $(1 + \tau_c)c$ including consumption taxes, savings into the next period a^+, contributions to social security $T_p(\cdot)$ as well as progressive

[8]We refer to a household with more than one member as a family. Families can take the form of single-mothers, couples and couples with children.

[9]Each member of the household is given an equivalence value: 1.0 to the first adult, 0.5 to the second and 0.3 to each child. We don't distinguish between young and old children.

[10]Notation: In what follows, we use variables without a subscript to denote household aggregates, like aggregate consumption expenditure c. Variables with a subscript refer to individual level units, like individual consumption c_g. We apply the same notation to earnings y, pensions p, bequests b, etc.

[11]Intergenerational transfers consist only of accidental bequests that households might leave if they die before the terminal age J. We assume that the total of those accidental bequests is distributed lump-sum to all working age households.

income taxes $T(\cdot)$. Consequently, the household budget constraint reads

$$(1+\tau_c)c + a^+ + T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big) + T\Big(y_m, y_f, p, i\Big) \\ = (1+r)a + y + y_{mini} + p + t(k,i) + b. \quad (8)$$

Single households, of course, receive only one labor income, so that either y_m or y_f is equal to zero. Note that pension contributions are collected on a per person basis, while income taxation depends on family status, see below.

3.4 Dynamic Optimization Problems

Singles The current state of a household with a single adult person is described by a vector $\mathbf{x_s} = (j, g, s, \eta, h, \xi, k, a, e)$ that summarizes the household's age j, gender g, education s, her current labor productivity shock η, her labor market flexibility h, her employment costs ξ, the presence and age of kids k, her wealth position a as well as previously collected pension entitlements e. The dynamic optimization problem of a single household reads

$$v(\mathbf{x_s}) = \max_{c,\, \ell \leq h,\, a^+ \geq 0,\, e^+} \; u(c, \ell, \xi) + \beta \psi^s_{j+1,g} E\Big[v(\mathbf{x_s^+}) \;\Big|\; \mathbf{x_s}\Big] \quad (9)$$

with $\mathbf{x_s^+} = (j+1, g, s, \eta^+, h^+, \xi^+, k^+, a^+, e^+)$. Households maximize (9) subject to the budget constraint (8), the accumulation equation for pension entitlements (14) as well as the laws of motion for children k, utility costs ξ, the labor choice set h, and labor productivity η. The result of this dynamic program are policy functions c, ℓ, a^+, and e^+ that all depend on the household's current state $\mathbf{x_s}$.

Couples The current state of a household with two married adults is described by a vector $\mathbf{x_c} = (j, s_m, s_f, \eta_m, \eta_f, h, \xi, k, a, e_m, e_f)$. It summarizes the joint household states age j, the labor market flexibility of the female partner h, the employment costs ξ, the presence and age of kids k, and household wealth a. In addition, it contains the individual specific education levels s_m, s_f, labor productivity shocks η_m, η_f, as well as the balance on individual pension accounts e_m, e_f for husband and wife, respectively. The dynamic optimization problem of a couple reads

$$v(\mathbf{x_c}) = \max_{\substack{c_m, c_f, \ell_m, \ell_f \leq h, \\ a^+ \geq 0,\, e_m^+, e_f^+}} \Big[u(c_m, \ell_m, \xi) + u(c_f, \ell_f, \xi)\Big] + \beta \psi^c_{j+1} E\Big[v(\mathbf{x_c^+}) \;\Big|\; \mathbf{x_c}\Big] \quad (10)$$

with $\mathbf{x_c^+} = (j+1, s_m, s_f, \eta_m^+, \eta_f^+, h^+, \xi^+, k^+, a^+, e_m^+, e_f^+)$. We provide an analytical derivation of the household's first-order conditions in Appendix A.1.

3.5 Technology

A continuum of identical firms produce a single good Y under perfect competition. They hire both capital K at price r and labor L at price w on competitive spot markets. Firms operate a constant returns to scale technology

$$Y = \Omega K^{\alpha} L^{1-\alpha}. \tag{11}$$

Ω denotes the aggregate level of productivity, whereas α is the elasticity of output with respect to capital. In the process of production, a fraction δ of the capital stock depreciates. Given the assumptions about competition and technology, we can safely assume the existence of a representative firm that takes prices as given and operates the aggregate technology in (11). In addition to employing factor inputs, the firm has to invest I_t into its capital stock. The law of motion for the capital stock reads

$$(1+n)K_{t+1} = (1-\delta)K_t + I_t. \tag{12}$$

We choose a depreciation rate of $\delta = 0.07$, which leads to a realistic investment to output ratio of 21%, see German Statistical Office (2020). We set the capital share in production at $\alpha = 0.3$ to obtain a capital-to-output ratio of three and normalize the technology level Ω such that the wage rate per efficiency unit of labor w is equal to 1. Finally, we assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

3.6 The Pension System

The pension system collects payroll taxes at rate τ_p on any earnings from regular employment y_g as well as on a share ϱ of mini-job earnings y_{mini}. Earnings are subject to payroll taxation up to a contribution ceiling equal to $2\bar{y}$. Pension contributions consequently are

$$T_p(y_g, y_{mini,g}) = \tau_p \times \min\left[y_p,\ 2\bar{y}\right] \quad \text{with } y_p = y_g + \varrho \times y_{mini},$$

where y_p denotes pension relevant earnings.

The pension system in the initial equilibrium is proportional.[12] In reward for contributing to the system, individuals earn an *annual insurance amount* on their taxable earnings. In addition, all mothers of young children ($k = 1$) receive a pension top-up $t_{child}(k) = 1$ in order to compensate them for their reduced earnings while raising children. Earnings plus top-up can't exceed the contribution limit. The annual insurance amount hence is given by

$$e_{annual} = \min\left[y_p + t_{child}(k),\ 2\bar{y}\right] \tag{13}$$

Individuals accumulate their annual insurance amounts over time to a lifetime insurance amount e according to

$$e^+ = e + e_{annual}. \tag{14}$$

[12] We discuss the different progressive pension systems that we analyze in the reform scenarios in Section 4.2.

Their pension p is then calculated from the lifetime insurance amount at retirement entry

$$p(e_{j_r}) = \kappa \times \frac{e_{j_r}}{j_r - 19} \quad \text{where} \quad e_{j_r} = \sum_{j=20}^{j_r-1} e_{annual,j}. \tag{15}$$

κ denotes the replacement rate of the pension system and $\frac{e_{j_r}}{j_r-19}$ is the lifetime average of annual insurance amounts.

The pension system runs on a pay-as-you-go basis. This means that in equilibrium, total annual pension contributions need to be equal to the total amount of annual pension payments. We fix the pension contribution rate at its statutory rate of $\tau_p = 0.187$ in 2017. The accrual rate for mini-job earnings is $\varrho = 0.80$. These choices result in a gross pension replacement of $\kappa = 0.41$, which is similar to the German gross replacement rate for the mean earner as reported by OECD (2021).

3.7 The Tax System and Government Expenditure

The government collects proportional taxes on consumption expenditure at rate τ_c and operates a progressive tax on labor earnings y_g and pension payments p. Earnings from mini-jobs y_{mini} are tax free. Individuals can deduct their pension contributions for the purpose of taxation, i.e. taxable earnings are

$$y_{tax,g} = y_g - \tau_p \min(y_g, \bar{y}_{max}).$$

The income tax function then reads

$$T(y_m, y_f, p, i) = \begin{cases} \mathcal{T}\Big(y_{tax,g} + p\Big) & \text{if } i = s \\ 2\mathcal{T}\Big(\frac{y_{tax,m}+y_{tax,f}+p}{2}\Big) & \text{if } i = c, \end{cases} \tag{16}$$

where $\mathcal{T}(\cdot)$ denotes the tax schedule. Couples enjoy tax benefits through the income splitting method, meaning that they are taxed based on their average household earnings, regardless of how these earnings are distributed across partners.

We employ the 2017 statutory German progressive income tax code as depicted in Figure 10 in Appendix B.6. In addition, we set the proportional consumption tax rate at $\tau_c = 0.16$ to balance the fiscal budget. Although consumption goods are regularly taxed at a rate of 19% in Germany, many goods (such as food, books and newspapers) are taxed at a lower rate. In our simulations, we assume that the consumption tax rates balances the fiscal budget on an annual basis.

Tax revenue is used to finance (wasteful) government spending G and child related transfers $t(k, i)$ to families with children. We fix (wasteful) government consumption at 19% of GDP in the initial equilibrium economy, see German Statistical Office (2020), and assume that it is fixed per capita. In 2017, parents received a child benefit of EUR 192 per child and month. Moreover, we let the government pay additional tax financed child support payments to single mothers, which mimic both alimony payments but also subsistence transfers in the real world. We set these monthly child support payments to EUR 576 per child. Appendix B.6 provides additional details.

3.8 Capital Markets, Trade and Equilibrium

We model a small open economy that freely trades capital and goods on competitive international markets. All private savings that are not employed by the domestic production sector are invested abroad at the international interest rate $\bar{r}$, see Appendix A.3 for further detail on the capital market equilibrium and trade. We assume that the government collects all accidental bequests and redistributes them in a lump-sum way among the surviving working-age population through the intergenerational transfer b. Given an international interest rate and the exogenous fiscal policy parameters, a *recursive competitive equilibrium* of this model is a set of household policy functions, a measure of households, optimal production inputs, factor prices, accidental bequests, a net foreign asset position and a trade balance that are consistent with individual optimization and market clearance. A formal definition of the equilibrium is available in Appendix A.3.

4 Simulation Results

In this section, we present simulation results from our quantitative model. To build confidence in the model results, we first investigate the properties of the initial equilibrium economy and compare it to the data. We then turn to counterfactual policy simulations, in which we introduce progressive components into the pension formula.

4.1 The Initial Equilibrium Economy

Overall, our simulation model is successful in replicating real-life data both on the macro and at the micro level.

The macroeconomy Table 2 summarizes central macroeconomic aggregates of the initial economy and compares them to the data for 2017. In reality, private savings are somewhat higher than the capital stock. However, a substantial part of these assets come from the top 1 percent wealth holders, a particular group that we do not include in our model. As a result, the German economy holds net foreign assets worth about 45 percent of GDP. In the model, we calibrated the discount factor β such that private savings cover total capital demand of firms.

Table 2: Macroeconomic aggregates

Variable	Value (HS/Col)	Data 2017
Private Assets	300.17	433.09
Capital Stock	300.00	305.24
Net Foreign Assets	0.17	44.25
Private Consumption	60.01	52.11
Government Consumption	19.00	19.84
Investment	21.00	20.96
Trade Balance	−0.01	7.09
Labor Tax Revenue net of Child benefits	10.10	8.35
Consumption Tax Revenue	9.68	8.74
Pension contribution rate (in %)	18.70	18.70
Total pension payments	12.09	9.15

Variables in percent of GDP if not indicated otherwise.

Data sources: PA: Alvaredo et al. (2022), CS: German Statistical Office (2020), PD, NFA: Deutsche Bundesbank (2022), PC, GC, I, TB, LTR, CTR: German Statistical Office (2020), Pension data: DRV Bund (2020),

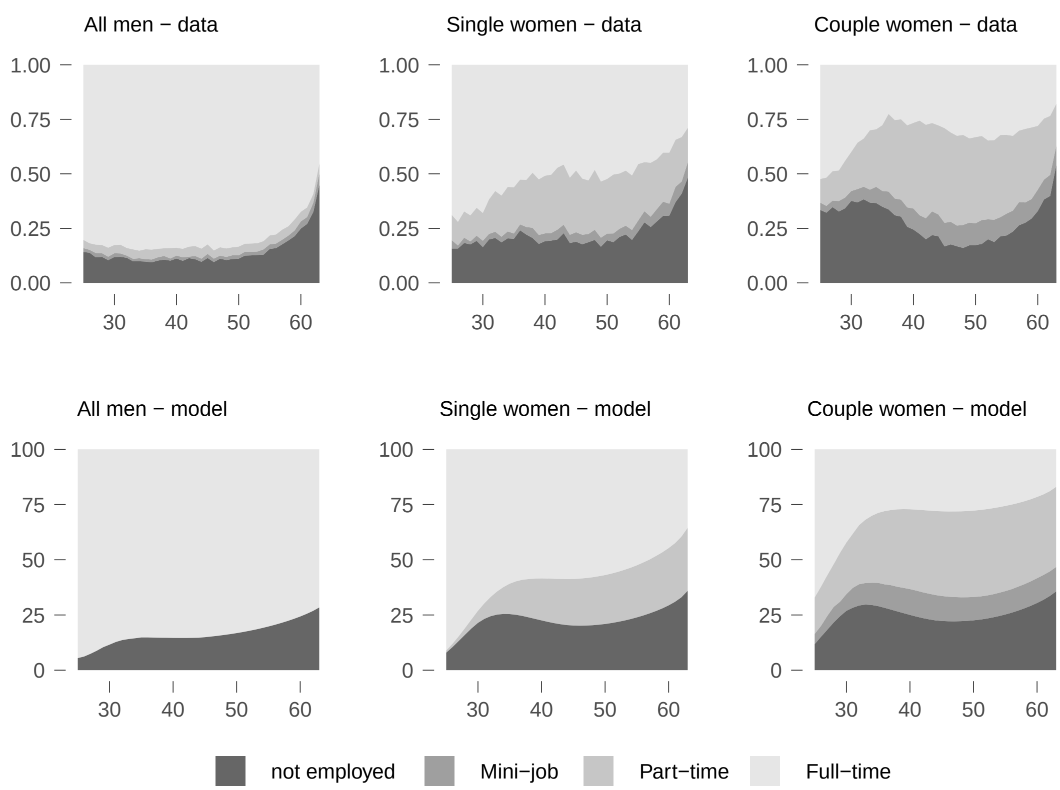
On the goods market, government consumption and investment match their empirical counterparts. The trade balance in our model is zero, like the net foreign asset position, which implies private consumption to be somewhat higher than in the data.[13] As the model neglects various other sources of taxation (like corporate taxes or property taxes), consumption and labor tax revenue are somewhat higher than in the data. By construction, we perfectly match the pension contribution rate. This leads to pension payments relative to GDP that are about a third higher than in reality. Note, however, that our model excludes civil servants or entrepreneurs who contribute to GDP but are not covered under the German public pension system.

Life-cycle labor supply profiles Figure 4 compares the life-cycle labor supply profiles of men and women living in different household arrangements from the data of the 2017 Microcensus (upper panel) to the model implied counterparts (lower panel). As we already noted before, non-employment rises with age and part-time work is virtually non-existent for men. Non-employment for women, on the other hand, exhibits a wave-shape over the life cycle, which is well replicated by the model. The wave-shape is predominantly driven by married mothers taking care of young children in their late 20s and 30s. Most importantly, the model

[13]Note that Germany has both a positive trade balance and a positive net foreign asset position. In a long-run equilibrium, this is impossible to achieve without a permanently positive balance of payments. Hence, we decided to strike a balance by having both the net foreign asset position and the trade balance equal to zero.

is able to match the quite distinct labor supply patterns of single and married women. For the latter, part-time and mini-job work is much more prevalent, even at older ages. Finally, the simulation model somewhat over-predicts the share of full-time workers at young ages for all demographic groups. This may result from the fact that liquidity constraints are still prevalent for young households and we may be missing a set of intergenerational transfers. In terms of our policy exercise, which aims at stimulating employment through the pension system, this means that we may underestimate the potential for labor supply responses somewhat for younger households.[14]

Figure 4: Empirical and simulated labor supply

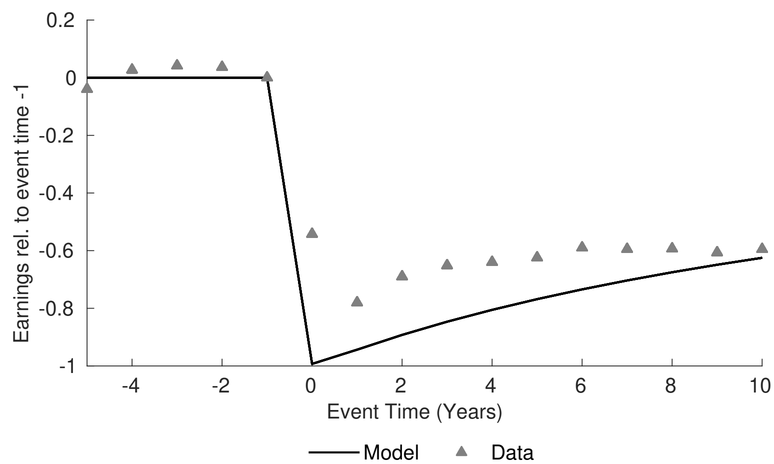


The motherhood penalty Figure 5 shows the motherhood penalty in our model and compares it to data from Kleven et al. (2019). To make a fair comparison, we perform an event study analysis in the same way as Kleven et al. (2019). In particular, we measure the evolution of women's labor earnings starting from the date they gave birth to their first child ($t = 0$), relative to their earnings in the previous period ($t = -1$). The results shown in the figure are an average over the entire model population, meaning that they represent single and married

[14]Yet, one may more generally ask whether we would expect the stimulative effect of pension reforms to be most prevalent in these age groups anyway.

women who give birth to children at different points in their life cycle. Like in the data, child birth is accompanied by a large drop in earnings. The initial drop in earnings is, most certainly, driven by changes in female labor hours. Over time, earnings somewhat recover, but even after ten years, a sizable 60 percent gap prevails. The long-run motherhood penalty then results from a mixture of lower working hours and the motherhood wage penalty, see Section 3.2. Overall, our model provides a decent fit for the empirical motherhood penalty. Naturally, the empirical estimates are somewhat fuzzy around the event date, as they are based annual earnings and women may give birth to children throughout the year. Yet, our model is especially successful in replicating the medium- and long-run effects on motherhood on earnings.

Figure 5: Empirical vs. simulated motherhood penalty



4.2 The Counterfactual: Annual vs. Lifetime Redistribution

In our counterfactual analysis, we replace the status-quo proportional German pension system with a progressive one. To this end, we draw on the progressive pension formula discussed in Figure 1 in the introduction as well as equation (3) in Section 2.

We use a medium range progressivity parameter of $\lambda = 0.5$.[15] In addition, we choose a bend point of $b = 0.3$. As we argue in another study, see Kindermann and Pueschel (2023), there is a tension in choosing this bend point: From the

[15]Simulation results for alternative progressivity parameters are available upon request. While different choices of λ result in different quantitative effects, their qualitative implications are identical.

perspective of economic theory, a small bend point is to be preferred, as it comes with the highest extensive margin labor supply incentives and therefore raises economic welfare. Yet in practice, having a bend point that is too small may lead workers to engage in very-low-hours or even fictitious working contracts just to benefit from the pension subsidy, which can again be detrimental to welfare. A value of $b = 0.3$ is a good compromise between these two objectives.

To ensure comparability between different simulations, we use the same set of structural parameters and fix per-capita government consumption at the initial equilibrium value. We assume that the contribution rate of the pension system remains at the initial equilibrium level as well. In doing so, we ensure that the size of the pension system relative to total labor hours is constant for all reforms. We use the replacement rate κ to balance the pension budget.[16] We compare results from long-run equilibrium outcomes only and neglect the transition path. This choice is based on the complexity and demand for physical space of the model. In Kindermann and Pueschel (2023) we demonstrate in a simpler version of this model that taking into account the short-run effects of progressive pension reforms typically enforces our argument.

Our focus is on two different types of redistributive pension systems, as already discussed in Section 1. The two systems redistribute pension income either on the basis of lifetime or annual earnings. In technical terms, this means that the progressive pension formula $f(\cdot)$ is applied at different stages of the accumulation process for pension claims.

Pension Reform: Lifetime Earnings Redistribution In a system with *lifetime earnings based redistribution*, we calculate the average of individual earnings over a lifetime and apply the progressive pension formula to this lifetime earnings index. This means that the calculation formula for the annual insurance amount (13) of an individual remains unchanged. Yet, the pension calculation formula (15), which computes pension payments from the accumulated lifetime insurances amount e_{j_r}, needs to be adjusted. Specifically, we set

$$e_{annual} = \min\left[y_p + t_{child}(k),\ 2\bar{y}\right] \quad \text{and} \quad p(e_{j_r}) = \kappa \times f\left(\frac{e_{j_r}}{j_r - 19}\right)$$

for the lifetime earnings based progressive pension.

Pension Reform: Annual Earnings Redistribution In a system with *annual earnings based redistribution*, we already apply the progressive pension formula to annual pension relevant earnings in order to calculate the annual insurance

[16]Note that, alternatively, we could fix total expenditure of the pension system at the initial equilibrium level. This is, however, counterfactual to the nature of a pay-as-you-go system. With fixed total expenditure, an increase in labor force participation or labor hours would lead to a decline in per capita pension payments and therefore lead to a cut in pension benefits which would counteract the positive effects of our pension reforms.

amount. Individuals then accumulate these insurance amounts over their working life to a lifetime index. Hence, this time the calculation formula for the annual insurance amount has to be adjusted and the pension calculation formula remains unchanged relative to the proportional pension system. Specifically, we set

$$e_{annual} = f\Big(\min\Big[y_p + t_{child}(k),\ 2\bar{y}\Big]\Big) \quad \text{and} \quad p(e_{j_r}) = \kappa \times \frac{e_{j_r}}{j_r - 19}$$

for the annual earnings based progressive pension.

Intuition In a very stylized way, the two different systems calculate the individual pension payment from either

$$\underbrace{p = \frac{\sum_{j=20}^{j_r} f(y_j)}{j_r - 19}}_{\text{annual earnings based}} \quad \text{or} \quad \underbrace{p = f\left(\frac{\sum_{j=20}^{j_r} y_j}{j_r - 19}\right)}_{\text{lifetime earnings based}}.$$

The rest of this paper is devoted to analyzing whether and how the choice of annual versus lifetime redistribution in a progressive pension system matters. Following the discussion in Section 2, we may expect a system with annual earnings redistribution to incentivize labor force participation for those individuals that are temporarily at the margin of non-participation. This includes workers with transitory low labor productivity shocks, those who are occupied by other tasks like caring for children, and those who approach retirement and have already built enough private savings to retire early. The participation incentives in a lifetime earnings based system, on the other hand, do not depend on instantaneous worker characteristics, but on (expected) lifetime average earnings only. Consequently, such a system may create the opposite type of incentives leading workers who are on the margin of non-participation to drop out of the labor force temporarily, as they will be compensated for a lack of earnings years through the progressive pension formula. On the positive side, a lifetime earnings based system may come at a more targeted level of redistribution between individuals permanently in need of resources and those who generally have enough to make a living on their own. Finally, in addition to setting participation incentives, both progressive systems will negatively distort the intensive margin labor supply decision for workers with medium range and high productivity, because they decouple the link between pension contributions and pension payments and lead to a higher implicit tax rate. To elaborate on all of these effects, we need a simulation model like the one presented above that has enough demographic detail to accurately quantify the labor supply responses and resource needs of a greatly heterogeneous population.

4.3 Labor Supply Effects

Table 3 shows the labor supply responses to the introduction of progressive pensions of the annual and the lifetime redistributive type, respectively.[17] The upper panel illustrates the results for the group of young workers, the lower panel for older workers. There are three patterns that immediately stand out: First, while employment increases for both age groups and both genders under the progressive pension with annual earnings redistribution, it unanimously falls under lifetime earnings redistribution. This is consistent with the argument made before that annual earnings redistribution provides positive participation incentives to those who are otherwise temporarily detached from the workforce or work in mini-jobs. Lifetime earnings redistribution, on the other hand, sets negative participation incentives for these workers, as they can expect to recover and to be compensated for missing earnings years through the progressive pension formula. Second, the introduction of progressive pensions generally distorts labor supply along the intensive margin, which leads to a lower number of full-time working women. Third, labor supply responses are larger for older workers. This is not surprising in light of that fact that older workers have a higher propensity to work at reduced hours or not at all. In addition, their accumulated savings make them more elastic to changes in the effective return to working. In terms of effect sizes relative to the initial equilibrium, the number of female workers who move from non-employment to employment under annual earnings redistribution is relatively small, in the order of 1 percent of the initially non-employed. Yet, the number of mini-job workers declines by a remarkable 11 and 26 percent for young and older women, respectively. On the other hand, the number of non-employed women increases by about 1.5 and 9 percent under lifetime earnings redistribution.

Table 3: Labor supply responses to progressive pensions by age groups

	Women				**Men**	
	not empl	mini-job	PT	FT	not empl	FT
			Young: Ages 25-44			
Annual ER	−0.16	−0.66	2.02	−1.20	−0.05	0.05
Lifetime ER	0.41	−0.18	0.54	−0.77	0.41	−0.41
			Older: Ages 44-63			
Annual ER	−0.41	−1.96	5.38	−3.00	−0.05	0.05
Lifetime ER	2.22	−0.75	2.05	−3.52	1.93	−1.93

Values in percentage points.

Labor supply reactions by family status are shown in Table 4. We can see the

[17]Refer to Table 7 in Appendix B for the initial distribution of workers over employment states.

same general patterns across the two systems with annual and lifetime earnings redistribution, respectively. Yet, we also find some important differences. The employment incentives of both systems are more pronounced for single individuals than for married couples. Single individuals are directly exposed to the benefits and costs of a changing degree of redistribution in the pension system. As such, they immediately react to positive work incentives as well as negative labor supply distortions. In couple households, decisions are made on a two-earner basis and, even if there are incentives to work an additional year, there may be good reasons not do so, for example because of a high degree of specialization. What is more, within a married couple, increased redistribution towards a secondary and/or low productive earner directly comes at the cost of declining pension entitlements for the primary and/or more productive earner. This constitutes a negative income effect for the entire family. Put differently, what comes as redistribution between different households for singles may only be redistribution within the family for married couples.

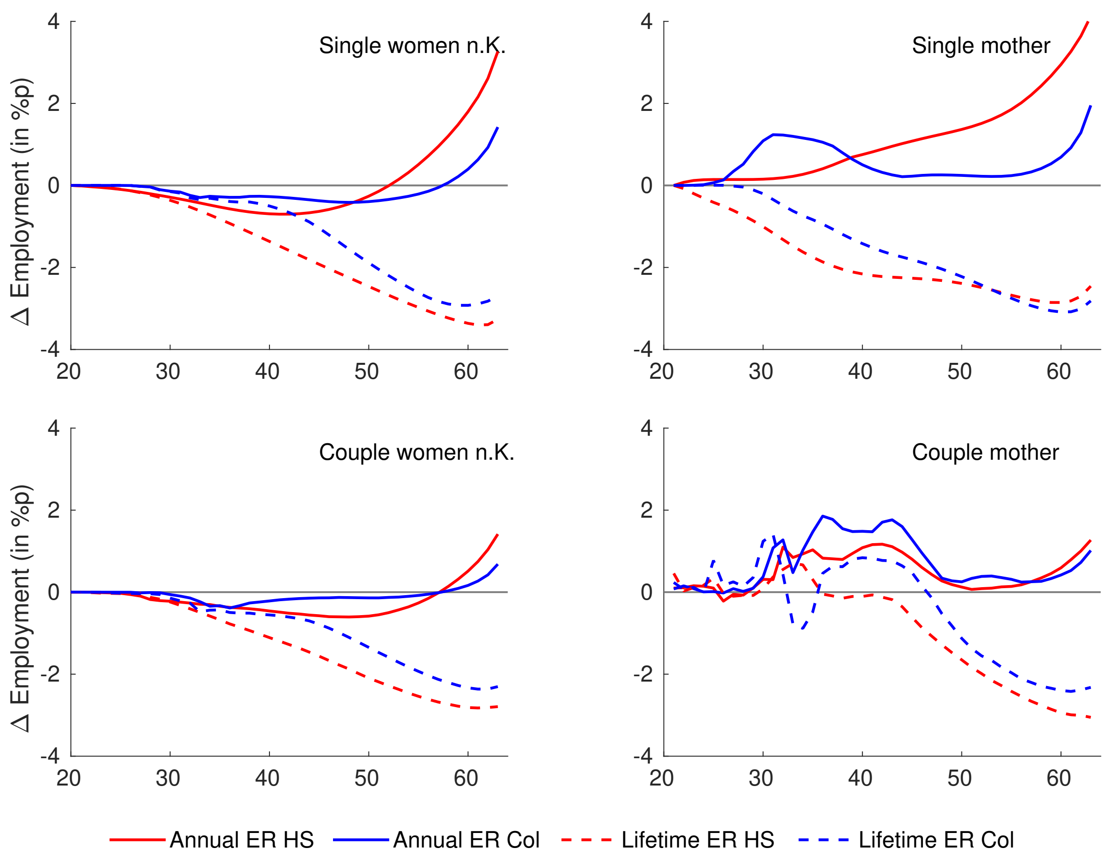
Table 4: Labor supply responses to progressive pensions by family status

	Women				**Men**	
	not empl	mini-job	PT	FT	not empl	FT
	Singles: Ages 25-63					
Annual ER	−0.38	0.00	2.42	−2.04	−0.14	0.14
Lifetime ER	1.72	0.00	1.33	−3.06	1.28	−1.28
	Couples: Ages 25-63					
Annual ER	−0.24	−1.84	4.15	−2.07	0.00	0.00
Lifetime ER	1.08	−0.65	1.24	−1.66	1.07	−1.07

Values in percentage points.

Figure 6 provides a deeper insight into the extensive margin employment effect over the entire life cycle for women with different education and family status. The solid lines indicate changes resulting from the introduction of an annual earnings based redistributive pension, while the dashed lines refer to a lifetime earnings based system. Red lines are for high school workers while blue lines refer to college graduates. Looking at these graphs, the employment incentives embedded in the annual and the lifetime redistribution systems seem even more clear cut. Women without children (left column) reduce their employment marginally between the ages of 20 and 50 to 55. This is owning to the fact that these women typically were full-time workers in the initial equilibrium and that the introduction of progressive pensions has diminished the return to working full time. At older ages, however, women without kids slowly start to reduce hours or even retire early when enough private savings are available. At this stage, the extra employment incentive set by an annual earnings based progressive pension immediately kicks in. As a result, the employment changes are significantly positive and can amount to almost four percentage points for women close to retirement. Mothers (right column) experience more positive employment effects over the entire life cycle. Mothers are typically working fewer hours or are not participating in labor activity at all, which is often related to the fact that they need to take care of children. For these women, the employment incentives are hence much more pronounced. Finally, the employment effects towards the end of the life cycle tend to be stronger for women with a high school degree than for college graduates, which can be explained by the different life-cycle productivity profiles of the two education groups, see Figure 3.

Figure 6: Changes in employment rates women



The picture looks quite different for a progressive pension in which redistribution is based on lifetime earnings. Here, the employment incentives are almost unanimously negative and the drop in employment becomes sharper as individuals approach retirement. Recall that incentives in such a system are purely based on lifetime average earnings and not on instantaneous earnings. Hence, as individuals approach retirement and they realize that their lifetime average earnings are high enough to bring them over the bend point $b\bar{y}$, their return to working diminishes and they may decide to drop out of the labor force. The only exception from this are married mothers with a college degree in their late 30s and 40s. To understand this, recall that they are most likely married to a college graduate husband who experiences a large drop in pension entitlements from the progressive pension reform. This negative income effect on the family may lead women to even increase their employment for some time to cover up for these losses.

The extensive margin responses of men, see Figure 13 in Appendix D, are in line with those of women. Again, an annual earnings based progressive pension sets positive employment incentives especially for older workers. Yet, the effects are generally smaller than for women. The lifetime earnings based redistribution scheme leads workers to drop out of the labor force especially at older ages. This is true for men of all education levels and family statuses.

4.4 Impact on the Macroeconomy

Table 5 shows the macroeconomic consequences of the two pension reforms. The introduction of progressive pensions reduces the risk of ending up with a too small amount of pension income at old age. Consequently, there is less need for precautionary behavior which lowers the overall amount of private savings. As argued before, the employment effects of the two different progressive pension systems are quite distinct. While an annual based redistributive system encourages employment, the lifetime earnings based system discourages it. This is reflected in the change of the employment-to-population ratio. Labor hours on the intensive margin (for women), on the other hand, are unanimously distorted downwards by both reforms. The effect on economy-wide labor input is a combination of changes in employment and changes in labor hours, weighted by the respective productivity of workers. Note that positive employment effects tend to kick in to a large degree for individuals with (temporarily) low productivity, while labor hours are distorted predominantly for high productive workers who were working on a full-time contract in the initial equilibrium. As a result, aggregate labor input – and therefore GDP and aggregate capital – fall in both reform scenarios. However, the drop in long-run GDP is by a factor 4 smaller when redistribution is based on annual earnings as compared to lifetime earnings. The decline in long-run GDP and the capital stock is accompanied by a reduction in aggregate consumption and investment, as it is typically found for fiscal reforms that come at increased redistribution and insurance.

On the fiscal side, the decline in aggregate labor input comes at a short-fall of labor tax revenue, which needs to be compensated by an increase in consumption taxes. Therefore, the consumption tax rate has to rise by 0.34 and 0.91 percentage points, respectively. Again, the fiscal consequences are much more moderate under a progressive pension with annual earnings based redistribution. Finally, aggregate pension payments relative to GDP fall in both reforms.

Table 5: Changes in macroeconomic aggregates

Variable	Annual ER	Lifetime ER
Private Savings	−1.23	−1.11
Employment-to-Population (in %p)	0.73	−0.87
Average Hours of the Employed (Women)	−0.31	−0.30
GDP/Capital Stock/Labor Input	−0.36	−1.59
Private Consumption	−0.61	−2.02
Investment	−0.36	−1.59
Consumption Tax Rate (in%p)	0.33	0.91
Consumption Tax Revenue	1.45	3.49
Labor Tax Revenue	−1.25	−3.00
Total pension payments	−0.41	−1.68

Changes over initial equilibrium in % if not indicated otherwise.

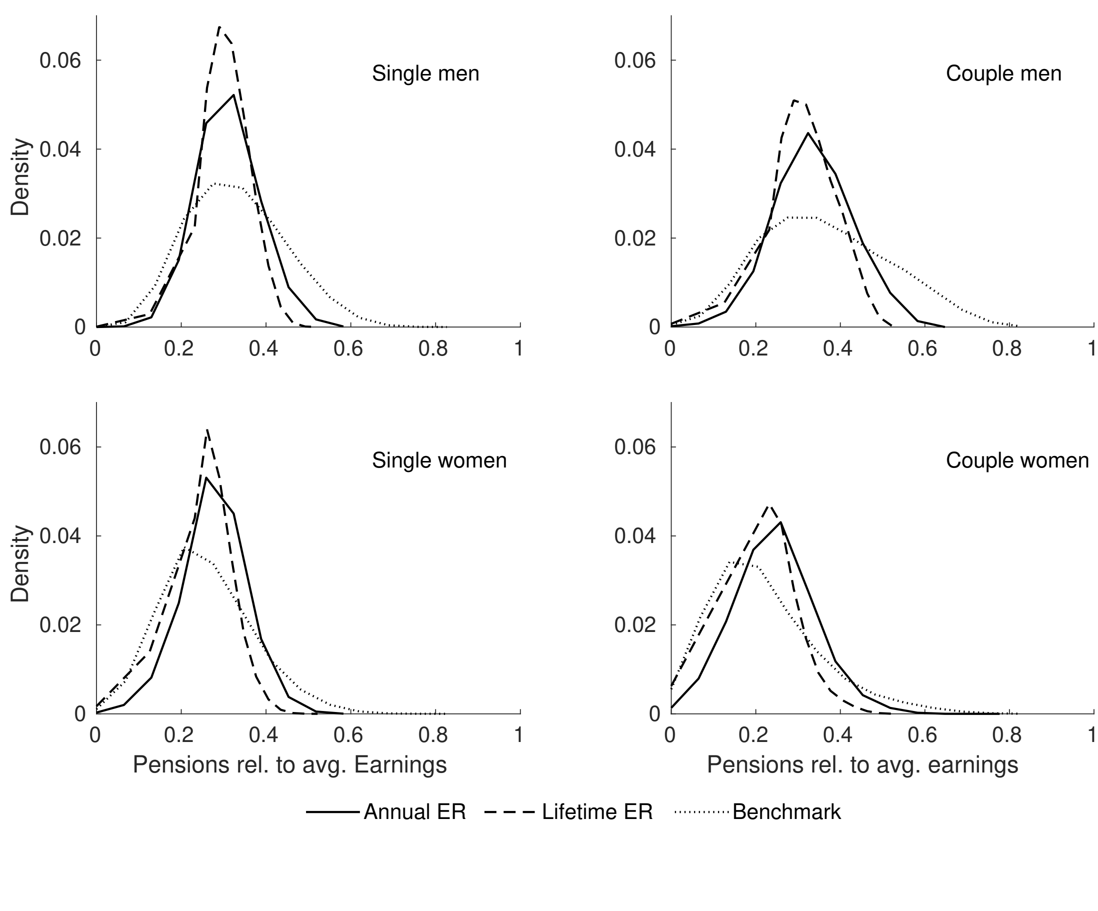
4.5 Distributional Effects

The primary purpose of introducing progressive pension systems into the economy is to reduced old-age income inequality. In this section, we therefore study changes in the distribution of pension income, but also in consumption and wealth.

The distribution of pension income Figure 7 shows the distribution of pension payments of individuals with different demographic characteristics at retirement entry relative to average labor earnings of the employed. The dotted lines indicate the situation under the proportional pension system in the initial equilibrium. The solid and dashed lines result from the introduction of progressive pension systems with annual and lifetime earnings based redistribution, respectively. The first thing that stands out is that both progressive pension reforms narrow the distribution of old-age pension income for both single and married individuals of all genders, which was their primary target. Yet, upon closer inspection, we also find important differences. For all household types but single men, the distribution of pension payments under the system with lifetime earnings distribution is shifted to the left relative to the system with annual earnings redistribution. For women in particular, the lifetime earnings based system hardly takes away any of the left tail of the distribution relative to the initial equilibrium. The annual earnings based system, on the other hand, reduces the size of the left tail and preserves a wider right tail of the old-age pension distribution.

The reason for this can again be found in the distinct employment effects of the two progressive pension systems. The annual earnings based system stimulates employment especially for workers who would otherwise only work in a mini-job or not at all. The additional working years paired with a sizeable earnings subsidy successfully reduces the very left tail of the old-age income distribution. Under the lifetime earnings based progressive pension, the very same workers rather drop out of the labor force (either temporarily or permanently) as they are (partly) compensated for missing years of contributions through the progressive pension formula. Yet, this compensation does not make up for the entire loss in pension entitlements from working a year less, which in turn leads to more mass on the left tail of the pension distribution. Summing up, a lifetime earnings based system would have the potential to create a more even distribution of pension payments by conditioning redistribution on permanent rather than transitory income. It fails to do so, however, because of its negative employment incentives on low productivity workers who end up desperately in need of funds at the end of their working life. In the end, keeping up work turns out to be a better insurance against the risk of old-age poverty than government redistribution.

Figure 7: Distribution of pension claims



4.5.1 The Variance of Consumption and Savings

Altering the distribution of old-age income triggers responses in labor supply but also in consumption and savings behavior. As a result, the distributions of consumption and private wealth change in response to progressive pension reforms. The left panel of Figure 8 shows the variance of log-consumption over the life cycle. The dotted line again marks the initial equilibrium, while the solid and the dashed lines indicate the two reform scenarios. The variance of consumption decreases markedly for all cohorts as a response to the reforms. While progressive pensions only change the distribution of income at old-age, households react to pension reforms by adjusting their savings behavior. In doing so, they can smooth the benefits of reduced old-age income inequality over their entire life cycle. The drop in the distribution of consumption is especially pronounced for the very old, who already ran down most of their wealth and solely rely on pension income to finance their consumption expenditure.

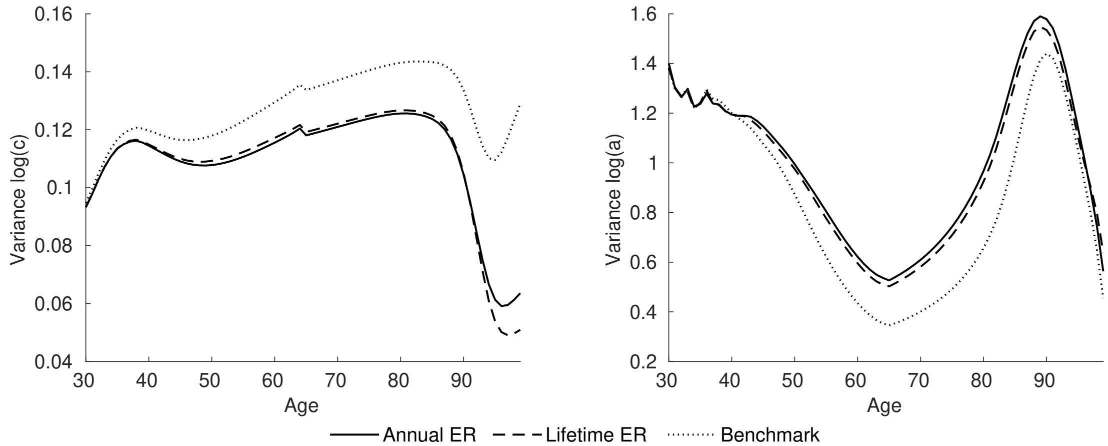
While the magnitude of the reduction in consumption inequality is similar across the two reform scenarios, there are still qualitative differences. A progressive pension with annual earnings redistribution is more successful in reducing consumption inequality during working life, while the lifetime earnings based system performs better at very old ages. This is because of the positive employment incentive of annual earnings redistribution. Households are pulled into the workforce during periods of low productivity shocks. This in itself reduces gross earnings inequality and hence consumption inequality during working life. The situation flips for the very old cohorts, for which the distribution of pension payments is the only thing that matters. The lifetime earnings based system apparently is capable of reducing the inequality in pension payments even more than the annual earnings based system.

The right panel of Figure 8 displays the variance of the log of private wealth over the life cycle. The wealth gap widens substantially as a side effect of the reform. On the one hand, poor households have to provide less for retirement on their own because of increased insurance through the pension system. The earnings rich, on the other hand, have to increase their savings rate to compensate for a short-fall in pension income. This natural reaction to progressive pensions increases the variance of wealth starting from age 40 for the remainder of the individual life cycle.

4.6 Welfare Analysis

The pension reforms we propose come along with positive distributional consequences but at the cost of a generally weaker macroeconomic performance. In addition, they trigger different employment changes for different population subgroups. To evaluate the reforms from a normative point of view and to paint a more complete picture of their economic impact, we finally study their long-run welfare effects. To this end, we calculate ex-ante expected lifetime utility before any information about the household's characteristics has been revealed. We then compare two allocations: the calibrated initial equilibrium with a proportional pension system and a new long-run equilibrium with a reformed pension

Figure 8: Variance of consumption log(c)



system. To give the welfare numbers a meaningful interpretation, we calculate the consumption equivalent variation CEV between the two utility levels. The CEV indicates by how many percent we would have to increase or decrease the consumption level of households at each age and each potential state in the initial equilibrium in order to make them as well off as in a reform scenario with progressive pensions. A negative value for CEV indicates that a reform of the pension system deteriorates long-run welfare.

Table 6: Change in ex-ante long-run welfare

	Annual ER	Lifetime ER
Entire Population	**0.288**	**−0.577**
– Single Men	0.099	−0.533
– Single Women	1.216	0.598
– Married Men	−0.135	−1.047
– Married Women	0.251	−0.850

Welfare effects are reported as CEV over initial equilibrium in percent.

The first row of Table 6 shows ex-ante welfare effects for the population at large. While the introduction of an annual earnings based progressive pension increases long-run welfare by almost 0.3 percent, a lifetime earnings based progressive pension deteriorates welfare by almost 0.6 percent. Hence, the long-run difference in welfare between annual and lifetime earnings based redistribution amounts to a remarkable 0.9 percent of consumption equivalent variation. The central reason for this obviously lies in the positive employment effects triggered by the annual earnings based progressive pension. The stimulation of employment for low productivity workers limits the negative macroeconomic consequences of increased redistribution, as the size of the workforce expands. On top, it leads to a more favorable distribution of pension claims, as marginal workers decide to remain in the workforce and receive a pension subsidy. Under the lifetime earnings based progressive system, the very same workers would rather drop out of the labor force and rely on the progressive pension to solve their problems at retirement. The stimulation of employment, yet, works as a much better insurance against the risk of old-age poverty than the hope for government redistribution, and therefore annual earnings redistribution is to be preferred over lifetime earnings redistribution.

Starting from the second row of Table 6, we compare the welfare effects of different population subgroups across the two pension systems. Women, and particularly single women, are the main beneficiaries of a progressive pension with annual earnings redistribution. This is not surprising in light of the fact that especially mothers tend to be the ones who are both marginally attached to the workforce and at the risk of old-age poverty. The annual earnings based progressive pension manages to increase their old-age provision and bring them back into the work force. The effects are much stronger for single than for married women, as the latter experience redistribution within the family. While married women are encouraged to work and can accumulate disproportionately high pension claims in a progressive pension system, their husbands, who typically are primary earners, receive a cut in pension benefits. This internal redistribution of pension claims is reflected in the welfare numbers. Single women, on the other hand, purely benefit from redistribution across households, which rationalizes their much higher welfare numbers.

5 Conclusion

Should redistribution be based on annual or lifetime earnings? This paper makes a clear point in favor of annual earnings redistribution. We study the economic effects of the introduction of a progressive pension formula into the German economy, which currently features a proportional pension system. We thereby allow for two different scenarios: an annual earnings based progressive pension and a lifetime earnings based progressive pension.

We find that the annual earnings based progressive pension performs significantly better along several dimensions: it encourages employment of the productivity poor and the elder working-age population, it comes at a better overall macroeconomic performance and it leads to a superior distribution of pension payments at retirement. Most of these effects stem from the fact that under an annual earnings based progressive pension, individuals need to work in order to enjoy the benefit of redistribution. In contrast, under lifetime earnings redistribution, workers are even (partly) compensated for staying at home through the progressive pension formula. It turns out that working and enjoying a pension subsidy is much more fruitful in fighting the risk of old-age poverty than waiting for fiscal redistribution, as it improves both the individual and also the macroeconomic situation.

Of course, our analysis also comes with some limitations. One the one hand, we only study the welfare effects of pension policy across long-run equilibria, therefore neglecting potential transitional costs. In another study, however, we find in a simpler model that taking into account transition dynamics may even enforce welfare effects, see Kindermann and Pueschel (2023). The reason is, first an foremost, that private savings decline along the transition which gives individuals some additional consumption possibilities in the short-run. On the other hand, one may want to look at the joint determination of progressivity of the tax and pension system, like in Abraham et al. (2023). Owing to the complexity of the model, however, this will have to involve some simplifying assumptions.

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Lifetime Earnings Redistribution: Favourable or Costly?

Appendix for Online Publication

Fabian Kindermann and Veronika Püschel

A Simulation Model: Computational Details

In this Appendix, we describe further details of the quantitative simulation model. We first show the first order conditions of the household optimization problem. We then describe the stationary recursive competitive equilibrium of the economy and show how to compute the invariant measure of households. Finally, we briefly describe the computational algorithm.

A.1 The single household: First order conditions

In the following, we derive the solution of the single household's problem in an economy with a proportional pension system. Since labor supply constitutes a discrete choice, we can not formulate a first-order condition for labor supply. Instead, we solve the problem in two steps. First, we assume the household had already made a labor supply choice ℓ. Conditional on this labor supply decision, we determine the optimal consumption-saving decision by solving the conditional optimization problem $\tilde{v}(\mathbf{x_s}, \ell)$.

Consumption–savings choice: The dynamic household optimization problem reads

$$\tilde{v}(\mathbf{x_s}, \ell) = \max_{c_g, a^+} \frac{c_g^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{\left(\zeta_{k,s,g} + \ell\right)^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell>0} + \beta \psi^s_{j+1,g} E\left[v(\mathbf{x_s^+}) \middle| j, s, \eta, g, h, k, \ell \right]$$

with $\mathbf{x_s} = (j, g, s, \eta, h, k, \xi, a, e)$ and $\mathbf{x_s^+} = (j{+}1, g, s, \eta^+, h^+, k^+, \xi^+, a^+, e^+)$. Households maximize their conditional utility with respect to the budget constraint

$$(1 + \tau_c) \times c_g \times v(j, k, s) + a^+ + T_p(y, y_{mini}) + T\Big(y, 0, p, s\Big) = (1 + r)a + y + y_{mini} + p + t(k, i) + b. \quad (17)$$

The first-order conditions of the household then read

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c} &= c_g^{-\frac{1}{\sigma}} - \mu(1+\tau_c)\upsilon(j,k,s) = 0 \\
\frac{\partial \mathcal{L}}{\partial a^+} &= -\mu + \beta\psi_{j+1,g}^s E\left[v_a(\mathbf{x}_\mathbf{s}^+) \;\middle|\; j,s,\eta,g,h,k,\ell\right] = 0,
\end{aligned}
$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. Using the envelope theorem, we immediately obtain

$$v_a(\mathbf{x}_\mathbf{s}^+) = (1+r)\mu^+.$$

The Euler equation then reads

$$
\begin{aligned}
&\frac{c_g^{-\frac{1}{\sigma}}}{(1+\tau_c)\times\upsilon(j,k,s)} \\
&\qquad = (1+r)\beta\psi_{j+1,g}^s E\left[\frac{c_g^+(\mathbf{x}_\mathbf{s}^+)^{-\frac{1}{\sigma}}}{(1+\tau_c)\times\upsilon(j+1,k^+,s)} \;\middle|\; j,s,\eta,g,h,k,\ell\right]. \qquad (18)
\end{aligned}
$$

The Euler equation (18) and the budget constraint (17) define the optimal level of total household consumption $c(\mathbf{x}_\mathbf{s},\ell) = c_g \times \upsilon(j,k,s)$, savings $a^+(\mathbf{x}_\mathbf{s},\ell)$ and the utility value $\tilde{v}(\mathbf{x}_\mathbf{s},\ell)$ conditional on a certain labor supply decision $\ell \in \{0,\ell_{full}\}$ for single men and $\ell \in \{0,\ell_{mini},\ell_{part},\ell_{full}\}$ for female households.[18] Furthermore, the law of motion of pension entitlements

$$e^+(\mathbf{x}_\mathbf{s},\ell) = e + \min\left[y(\mathbf{x}_\mathbf{s},\ell) + \varrho \times y_{mini}(\mathbf{x}_\mathbf{s},\ell) + t_{child}(k),\ 2\bar{y}\right] \qquad (19)$$

determines the conditional pension entitlements $e^+(\mathbf{x}_\mathbf{s},\ell)$.

Labour supply decision: Given the utility $\tilde{v}(\mathbf{x}_\mathbf{s},\ell)$ for every possible $\ell \le h$, the utility maximizing labor supply decision is

$$\ell(\mathbf{x}_\mathbf{s},\xi) = \arg\max \tilde{v}(\mathbf{x}_\mathbf{s},\ell).$$

A.2 The couples household: First order conditions

The solution of the couple household's problem is quite similar. We first solve the consumption-saving problem conditional on the labor supply choices $\ell_m \in$

[18] Note that the female household might be constraint in her labor choice set, i.e. $\ell \le h$ must hold.

$\{0, \ell_{full}\}$ and $\ell_f \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$. For every possible and valid (ℓ_m, ℓ_f) combination we solve the optimization problem

$$\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f) = \max_{\substack{c_m, c_f, \\ a^+, e_m^+, e_f^+}} \Bigg[\frac{c_m^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_m \frac{(\zeta_{k,c,m} + \ell_m)^{1+\frac{1}{\chi_m}}}{1+\frac{1}{\chi_m}} - \xi \times \mathbb{1}_{\ell_m > 0} + \frac{c_f^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_f \frac{(\zeta_{k,c,f} + \ell_f)^{1+\frac{1}{\chi_f}}}{1+\frac{1}{\chi_f}} - \xi \times \mathbb{1}_{\ell_f > 0} \Bigg] + \beta \psi_{j+1}^c E\Big[v(\mathbf{x_c^+}) \;\Big|\; j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \Big]$$

with

$$\mathbf{x_c} = (j, s_m, s_f, \eta_m, \eta_f, k, h, \xi, a, e_m, e_f) \quad \text{and}$$
$$\mathbf{x_c^+} = (j+1, s_m, s_f, \eta_m^+, \eta_f^+, k^+, h^+, \xi^+, a^+, e_m^+, e_f^+).$$

Couples maximize their utility subject to the borrowing constraint $a^+ \geq 0$, and the budget constraint

$$(1+\tau_c) \times (c_m + c_f) \times \upsilon(j,k,c) + a^+ + T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big) + T\Big(y_m, y_f, p, i\Big) = (1+r)a + y + y_{mini} + p + t(k,i) + b. \quad (20)$$

The first-order conditions read

$$\frac{\partial \mathcal{L}}{\partial c_g} = c_g^{-\frac{1}{\sigma}} - \mu(1+\tau_c)\upsilon(j,k,c) = 0$$
$$\frac{\partial \mathcal{L}}{\partial a^+} = -\mu + \beta \psi_{j+1}^c E\Big[\tilde{v}_a(\mathbf{x_c}, \ell_m, \ell_f) \;\Big|\; j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \Big] = 0,$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. We immediately see that $c_m = c_f$ needs to hold. Using the envelope theorem, we obtain

$$v_a(\mathbf{x_c^+}) = (1+r)\mu^+.$$

The Euler equation then reads

$$\frac{c_g^{-\frac{1}{\sigma}}}{(1+\tau_c) \times \upsilon(j,k,c)} = (1+r)\beta\psi_{j+1}^c E\left[\frac{c_g^{-\frac{1}{\sigma}}(\mathbf{x_c^+})}{(1+\tau_c) \times \upsilon(j+1,k^+,c)} \;\middle|\; j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \right]. \quad (21)$$

The Euler equation (21) and the budget constraint (20) define the optimal level of total household consumption $c(\mathbf{x_c}, \ell_m, \ell_f) = (c_m + c_f) \times \upsilon(j,k,c)$, savings

$a^+(\mathbf{x_c}, \ell_m, \ell_f)$ and the utility value $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ conditional on a certain labor supply decision (ℓ_m, ℓ_f). Furthermore, the laws of motion for pension entitlements

$$e_g^+(\mathbf{x_c}, \ell_g) = e_g + \min\left[y_g(\mathbf{x_c}, \ell_g) + \varrho \times y_{mini,g}(\mathbf{x_c}, \ell_g) + t_{child}(k),\ 2\bar{y}\right] \tag{22}$$

determines the conditional pension entitlements for each partner $e_g^+(\mathbf{x_c}, \ell_g)$.

Labour supply decision: Given the conditional utility $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ for every possible (ℓ_m, ℓ_f) combination with $l_f \leq h$, the household chooses the utility maximizing labor supply combination

$$\Big(\ell_m(\mathbf{x_c}), \ell_f(\mathbf{x_c})\Big) = \arg\max \tilde{v}(\mathbf{x_c}, \ell_m, \ell_f).$$

A.3 Stationary Recursive Competitive Equilibrium

Definition 1. *Given an international interest rate $\bar{r}$, government expenditures G, a consumption tax rate τ_c, a progressive tax system $T(\cdot)$ as well as a characterization of the pension system $\{\tau_p, \varrho, \kappa\}$, a stationary recursive equilibrium with population growth n is a collection of value and policy functions $\{v, c, \ell, a^+, ep^+\}$ for a single and $\{v, c, \ell_m, \ell_f, a^+, e_m^+, e_f^+\}$ for a couple household, optimal production inputs $\{K, L\}$, accidental bequests $\{b_j\}_{j=1}^J$, a net foreign asset position and a trade balance $\{Q, TB\}$ as well as factor prices $\{r, w\}$ that satisfy*

1. *(Household Optimization) Given prices and characteristics of the tax and pension system, the value function v satisfies the Bellman equation* (9) *for singles and* (10) *for couples together with the budget constraint* (8)*, the accumulation equations for pension claims* (13) *and* (14)*, the borrowing constraint $a^+ \geq 0$ and the laws of motion for productivity risk, career choice and fertility. c, ℓ, a^+ and e^+ are the associated policy functions for singles and $c, \ell_m, \ell_f, a^+, e_m^+, e_f^+$ the policy functions for couples.*

2. *(Firm Optimization) Given the international interest rate $\bar{r}$ as well as the wage rate w, firms employ capital and labor according to the demand functions*

$$\bar{r} = \Omega\alpha\left(\frac{L}{K}\right)^{1-\alpha} - \delta \ \ \text{and} \ \ w = \Omega(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

   Aggregate output is calculated from (11) *and the capital stock evolves according to* (12)*.*

3. *(Government Constraints) The budget constraint of the pension system*

$$\underbrace{\int p(e_{j_r}) \times \mathbb{1}_{j \geq j_r}\ d\Phi^s + \int \left[p(e^m_{j_r}) + p(e^f_{j_r})\right] \times \mathbb{1}_{j \geq j_r}\ d\Phi^c}_{\textit{total pension claims}} =$$

$$\underbrace{\int T_p\Big(y_g, y_{mini,g}\Big)\ d\Phi^s + \int T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big)\ d\Phi^c}_{\textit{total contributions}}. \quad (23)$$

and that of the tax system

$$\tau_c \times C + \int T\Big(y_g, 0, p, s\Big)\ d\Phi^s + \int T\Big(y_m, y_f, p, c\Big)\ d\Phi^c$$
$$= G + \int t(k, s)\ d\Phi^s + \int t(k, c)\ d\Phi^c \quad (24)$$

hold. Accidental bequests are calculated from

$$b_{j,t} = \frac{\int \frac{1-\psi^s_{j,g}}{\psi^s_{j,g}} \times (1+r)a\ d\Phi^s_t + \int \frac{1-\psi^c_j}{\psi^c_j} \times (1+r)a\ d\Phi^c_t}{\int \mathbb{1}_{j<j_R}\ d\Phi^s_t + \int \mathbb{1}_{j<j_R}\ d\Phi^c_t} \quad \textit{if } j < j_R. \quad (25)$$

4. *(Market Clearing:)*

   (a) *The labor market clears:*

$$L = \int w(j, s, \eta, g, k, \ell) l(\boldsymbol{x}_s)\ d\Phi^s + \int w(j, s, \eta, g, k, \ell) l(\boldsymbol{x}_c)\ d\Phi^c$$

   (b) *The capital market clears:*

$$K + Q = \int a\ d\Phi^s + \int a\ d\Phi^c$$

   (c) *The balance of payments identity is satisfied:*

$$TB = (n - \bar{r})Q$$

   (d) *The goods market clears:*

$$Y = \int c(\boldsymbol{x}_s)\ d\Phi^s + \int c(\boldsymbol{x}_c)\ d\Phi^c + (n+\delta)K + G + TB.$$

5. *(Consistency of Probability Measure Φ) The invariant probability measure is consistent with the population structure of the economy, with the exogenous processes of labor productivity η, labor flexibility h and fertility k, and the household policy functions a^+ and e^+ or e^+_m and e^+_f, respectively. A formal definition of the probability measures is provided in Appendix A.4.*

A.4 The Measure of Households

The population consists of couple and single households which operate on different state spaces. At age 20, the mass of couple households Φ^c and the mass of single households Φ^s sum to one.

Couple households At age 20, couple households draws one of four possible education level (s_m, s_f) from the joint distribution $\phi_s^c(s_m, s_f)$. Conditional on the their education level, each partner draws an initial labor productivity η from the invariant distribution $\pi_{\eta,20}$ of the process for η. The household enters the economy without kids $k = 0$, the full labor choice set $h = \ell_{full}$, zero assets $a = 0$ and zero pension claims $e_m = e_f = 0$. The realization of ξ follows a log-normal distribution $\pi_\xi(\xi)$ with mean μ_ξ and variance σ_ξ^2. Thus, the measure of couple households with characteristics $\mathbf{x_c} = (s_m, s_f, \eta_m, \eta_f, h, \xi, k, a, e_m, e_f)$ is constructed as

$$\Phi^c(\{20\}, \{s_m\}, \{s_f\}, \{\eta_m\}, \{\eta_f\}, \{\ell_{full}\}, \{\xi\}, \{0\}, \{0\}, \{0\}, \{0\}) = \\ \phi_c \times \phi_s^c(s_m, s_f) \times \pi_{\eta_m,20}(\eta_m \mid s_m) \times \pi_{\eta_f,20}(\eta_f \mid s_f) \times \pi_\xi(\xi),$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and pension claims of the husband $\mathcal{EP}_m$ and the wife $\mathcal{EP}_f$ we have

$$\Phi^c(\{j+1\}, \{s_m\}, \{s_f\}, \{\eta_m^+\}, \{\eta_f^+\}, \{h^+\}, \{\xi^+\}, \{k^+\}, \mathcal{A}, \mathcal{EP}_m, \mathcal{EP}_f) = \\ = \frac{\psi_{j+1}^c \times \pi_\eta(\eta_m^+ \mid \eta_m, s) \times \pi_\eta(\eta_f^+ \mid \eta_f, s) \times \pi_h(h^+ \mid h, f, \ell) \times \pi_\xi(\xi) \times \pi_k(k^+ \mid k, j, c)}{1+n} \\ \times \int \mathbb{1}_{\{a^+(\mathbf{x_c}) \in \mathcal{A}\}} \times \mathbb{1}_{\{e_m^+(\mathbf{x_c}) \in \mathcal{EP}_m\}} \times \mathbb{1}_{\{e_f^+(\mathbf{x_c}) \in \mathcal{EP}_f\}} \\ \Phi^c(\{j\}, \{s_m\}, \{s_f\}, \{\eta_m\}, \{\eta_f\}, \{h\}, \{\xi\}, \{k\}, da, de_m, de_f),$$

where the integral is the measure of assets a and pension claims e_m and e_f today such that for fixed $(j, s_m, s_f, \eta_m, \eta_f, h, \xi, k, \ell_f)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_c})$ lies in $\mathcal{A}$ and the optimal choice today of pension claims for tomorrow $e_m^+(\mathbf{x_c})$ and $e_f^+(\mathbf{x_c})$ lie in $\mathcal{EP}_m$ and $\mathcal{EP}_f$, respectively.

Single households Next, we construct the measure of single households across the characteristics $\mathbf{x_s} = (g, s, \eta, h, \xi, k, a, e)$. At age 20, households draw a gender $g \in \{0, 1\}$ and an education level $s \in \{0, 1\}$, where $g = 1$ occurs with probability ϕ_g and $s = 1$ with probability ϕ_s^g. Conditional on the education level, households draw an initial labor productivity η from the distribution $\pi_{\eta,20}$, see above. Households enter the economy without kids, the full labor choice set $h = \ell_{full}$, zero assets and zero pension claims. Thus,

$$\Phi^s(\{20\}, \{g\}, \{s\}, \{\eta\}, \{\ell_{full}\}, \{\xi\}, \{0\}, \{0\}, \{0\}) \\ = \phi_s \times \phi_g(g) \times \phi_g^s(s_g) \times \pi_{\eta,20}(\eta \mid s) \times \pi_\xi(\xi),$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and pension claims $\mathcal{EP}$ we have

$$\Phi(\{j+1\},\{g\},\{s\},\{\eta^+\},\{h^+\},\{\xi^+\},\{k^+\},\mathcal{A},\mathcal{EP}) =$$
$$= \frac{\psi^s_{j+1,g} \times \pi_\eta(\eta^+ \mid \eta, s) \times \pi_h(h^+ \mid h, g, \ell) \times \pi_\xi(\xi) \times \pi_k(k^+ \mid k, j, s)}{1+n}$$
$$\times \int \mathbb{1}_{\{a^+(\mathbf{x_s})\in\mathcal{A}\}} \times \mathbb{1}_{\{e^+(\mathbf{x_s})\in\mathcal{EP}\}}\ \Phi(\{j\},\{s\},\{\eta\},\{h\},\{\xi\},\{k\},da,de)$$

where the integral is the measure of assets a and pension claims e today such that, for fixed $(j, g, s, \eta, h, \xi, k\ell)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_s})$ lies in $\mathcal{A}$ and the optimal choice today of pension claims for tomorrow $e^+(\mathbf{x_s})$ lies in $\mathcal{EP}$.

A.5 Computational Algorithm

Following Kindermann et al. (2020), we solve the model in two steps. First, we apply the method of endogenous grid points to solve the household's consumption-savings problem. We can then compute policy functions $c(\mathbf{x_s}), \ell(\mathbf{x_s}), a^+(\mathbf{x_s})$ for single households and $c(\mathbf{x_c}), \ell_m(\mathbf{x_c}), \ell_f(\mathbf{x_c}), a^+(\mathbf{x_c})$ for couple households as well as the value functions $v(\mathbf{x_s})$ and $v(\mathbf{x_c})$. Second, we determine equilibrium quantities and prices using a standard rootfinding method.

A.5.1 Computation of Policy and Value Functions

This section presents the method for computing the policy and value functions of single households using the method of endogenous gridpoints. The solution method for couple households is technically identical. The state of a single household is given by $\mathbf{x_s} = (j, g, s, \eta, h, \xi, k, a, e)$. To solve the model on a computer, we start with discretizing the continuous elements a, e, η. We use routines provided by the toolbox that accompanies Fehr and Kindermann (2018).

- We specify the asset grid $\hat{\mathcal{A}} = \{\hat{a}_0, \ldots, \hat{a}_{40}\}$ as nodes with growing distance on the interval $[\bar{a}_l, \bar{a}_u]$. In particular, we let

  $$\hat{a}_i = \bar{a}_l + \frac{\bar{a}_u - \bar{a}_l}{(1+g_a)^{40} - 1} \times [(1+g_a)^i - 1] \text{ for } i = 0, 1, \ldots, 40.$$

  The lower limit of the asset grid is $\bar{a}_l = 0$, the upper limit of the asset grid is $\bar{a}_u = 50$, the growth rate of gridpoints is $g_a = 0.14$.

- We specify the earnings points grid $\widehat{\mathcal{EP}} = \{\hat{ep}_0, \ldots, \hat{ep}_{12}\}$ as a grid on the interval $[0, 2]$ with equally spaced nodes.

- We approximate the stochastic process of the AR(1) labor productivity process η by a discrete Markov chain. We use the Rouwenhorst method to discretize the stochastic process $\hat{\mathcal{E}} = \{\hat{\eta}_1, \dots, \hat{\eta}_5\}$ and to determine a transition matrix

$$\pi_\eta(\eta^+|\eta) = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{15} \\ \pi_{21} & \pi_{22} & \dots & \pi_{25} \\ \vdots & \vdots & \ddots & \dots \\ \pi_{51} & \pi_{52} & \dots & \pi_{55} \end{bmatrix}. \tag{26}$$

The policy and value functions can now be solved via backward induction. In the last possible age J, the household will not work[19] and not save, but will consume all remaining resources. This determines the policy functions

$$\begin{aligned} c(J, s, \hat{\eta}_g, h, \xi, k, \hat{a}_i, \hat{e}_k) &= \frac{(1+r)\hat{a}^i + p(\hat{e}_k) - T(0,0,p,s) + b}{(1+\tau_c) \times \upsilon(J,k,s)}, \\ l(J, s, \hat{\eta}_g, h, \xi, k, \hat{a}_i, \hat{e}_k) &= 0, \\ a^+(J, s, \hat{\eta}_g, h, \xi, k, \hat{a}_i, \hat{e}_k) &= 0 \end{aligned}$$

and the value function

$$v(J, s, \hat{\eta}_g, h, \xi, k, \hat{a}_i, \hat{e}_k) = \frac{c(J, s, \hat{\eta}_g, h, \xi, k, \hat{a}_i, \hat{e}_k)^{1-\sigma}}{1-\sigma}$$

for all $i = 0, \dots, 40,\ k = 0, \dots, 15,\ g = 0, \dots, 5.$

With the final period policy functions and value function at hand, we can iterate backwards over ages to determine the full history of household decisions. We now only describe the procedure for working-age single households. The optimization problem as well as the first-order conditions for these households are discussed in Appendix A.1.

We now apply the method of endogenous gridpoints. We use the exogenous grid $\hat{\mathcal{A}} = \{\hat{a}_i\}_{i=0}^{40}$ to indicate the remainder of assets into the next period, i.e. $a^+ = \hat{a}_v$. For each state $\tilde{x} = (j, s, \eta, h, k, \mathbf{a}^+, e)$ and possible labor choice $\ell \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$:

1. we first determine

$$e^+ = \frac{(j-1)e}{j} + \frac{\min\left(y + y_{mini},\ 2\bar{y}\right)}{j}$$

2. given a^+ and e^+ we determine $c(\tilde{x})$ from the Euler Equation (18)

3. with $l(\tilde{x})$ and $c(\tilde{x})$, we use the budget constraint (17) to get $a(\tilde{x})$.

[19]Remember, the compulsory retirement age is j_r.

Once $l(\tilde{x})$, $c(\tilde{x})$ and $a(\tilde{x})$ are solved, we can interpolate along a to obtain the policy functions $l(\mathbf{x_s})$, $c(\mathbf{x_s})$ and $a^+(\mathbf{x_s})$ as well as the value function $v(\mathbf{x_s}$ for each today's asset value $\hat{a}_i,\ i = 0, \ldots, 40$ and earnings points amount $\hat{e}_k, k = 0, \ldots, 12$ by piecewise linear interpolation.

In case the asset restriction $a^+ \geq 0$ is binding, we extend the interpolation data by another point of value 0 on the left and determine the policy and value functions at this point. We assume the household consumes all available resources and has no savings left over for tomorrow.

A.5.2 A long-run equilibrium of the macroeconomy

We model a small open economy, hence prices r and w are fixed. In order to determine aggregate quantities and policy parameters in the initial equilibrium ($t = 0$) we need to determine the following four variables numerically:

- the government budget balancing consumption tax rate τ_c that satisfies equation (24)
- the pension replacement rate κ that balances pension contributions and pension payments as outlined in equation (23)
- average earnings $\bar{y}_t$ of the employed population[20]
- aggregate bequests $\bar{B}$, which immediately allows us to compute cohort bequests $\{b_j\}_{j=1}^{J}$, see equation (25).

Once a guess of these four variables is available, we can use the following algorithm to compute the remainder individual and aggregate variables of the economy:

1. We solve the household optimization problem using the guesses for $\tau_c, \kappa, \bar{y}, \bar{B}$ and determine the measure of households.
2. We compute aggregate quantities $\{L, K, Q, A, TB, Y, C, G, I, \Omega, B\}$ from individual decisions and the measure of household and determine the gap $D = Y - C - I - G$ between demand and supply.

We determine the four central parameters $(\tau_c, \kappa, \bar{y}_t, \bar{B})$ by means of a quasi-Newton rootfinding method. The method receives an initial guess of these variables and updates them in each iteration step using the (numerical) Jacobian of the determining equation system. The iteration process stops when the government and the pension budget are in equilibrium and the model implied average earning and aggregate bequest equal the guess provided by the method.

[20]This is an important parameter, as it determines the pension contribution cap, pension payments and earnings of the low-earning group.

B The Calibration Process in Detail

This appendix discusses our choices of functional forms and parameters in detail. We pay particular attention to demographics and the labor market characteristics of different types of households. We parameterize the model using data and calibration targets from the German economy, which currently features a proportional pension system. Germany therefore serves as a good benchmark for reforms that aim at introducing progressivity into the pension formula.[21] Our base year is 2017, in which the average contributory earnings – the empirical counterpart to average earnings $\bar{y}$ in our model – amounted to EUR 37,000, see DRV Bund (2020). Our parameterization process is a two-step procedure: We try to identify as many parameters in the model as possible to which we can directly assign values either by estimating them from data or by using direct estimates from the literature. We then use the remaining set of parameters to calibrate the model to data moments, which we again derived from micro and macro data of the German economy. We provide a summary table of all parameters at the end of this appendix.

B.1 Demographics

We let households start their economic life at the age of 20 and allow for a maximum life span of 99 years. The mandatory retirement age j_r is 64, which equals the average retirement age of the German regular retirement population in 2017, see DRV Bund (2019).

B.1.1 Education and Marital Distribution

We use data from the age cohorts 35-49 of the 2017 German Microcensus[22] to estimate the following demographic parameters:

1. 50.78% of individuals in the sample are male, 33.02% of them have a college education, and 67.70% of them live in a couple household.[23]
2. The proportion of women is 49.22% and 27.70% of them have a college education.
3. We also examine patterns of assortative mating and find that 85.69% of non-college educated men are married to a non-college educated woman. 54.81% of college-educated men are married to a college-educated woman.

[21] U.S. Social Security would not make the best possible starting point, as it redistributes heavily towards single-earner married couples through spousal and survivor benefits, see for example Kaygusuz (2015). The German system has survivor benefits, too, but no spousal transfers.

[22] See RDC of the Federal Statistical Office and Statistical Offices of the Federal States (2021)

[23] This includes individuals who live in a couple household but are not formally married.

Informed by these empirical facts, we set the demographic parameters of the model as follows:

1. A share $\phi_m = 0.5078$ of each new-born cohort are men and $\phi_{c,m} = 0.6770$ of them are married, the remainder is single. For women, these shares are $\phi_f = 0.4922$ and $\phi_{c,f} = 0.6887$.
2. The education distribution within couples is given by

$$\phi_s^c(s_m, s_f) = \begin{bmatrix} \phi_s^c(0,0) = 0.5740 & \phi_s^c(0,1) = 0.0958 \\ \phi_s^c(1,0) = 0.1492 & \phi_s^c(1,1) = 0.1810 \end{bmatrix}, \tag{27}$$

   where $\phi_s^c(0,0)$ indicates that both partners are non-college educated and $\phi_s^c(1,1)$ that both are college educated. A single woman is college-educated with likelihood $\phi_s^f = 0.2774$ and a single men with likelihood $\phi_s^m = 0.3302$.

This calibration strategy provides us with a model consistent measure of households that matches the empirical targets.

B.1.2 Retirement and Survival

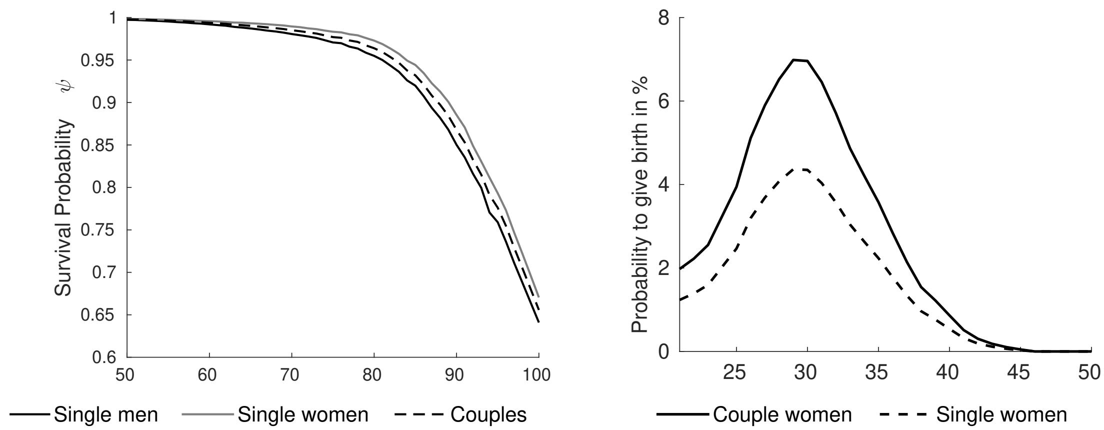
We extract the 2017 annual life tables for men and women from the Human Mortality Database (2020) to calculate average survival probabilities $\psi_{j,g}^i$ by gender. For couples, we simply take the average of the survival probabilities for both genders, i.e. $\psi_{j,m}^c = \psi_j^c = 0.5 \times (\psi_{j,m}^s + \psi_{j,f}^s)$. Single men die at age 79.5 on average, single women at age 84.1 and couples at age 81.7. The left panel of Figure 9 shows the respective survival probability profiles starting from age 50.

B.1.3 The Arrival and Presence of Children

In our model, women may give birth to two children at some date between the ages 21 and 45, i.e. their fertile life cycle. According to the 2017 German Microcensus, 80.02% of married women and 47.53% of single women in the cohort 35-49 had at least one child present in their household. We use these numbers as proxies for the overall likelihood of giving birth to children. Furthermore, we extracted data from Eurostat (2023) on mothers' age at first birth. Together these data lead us to the calibrated life-cycle child-birth probabilities $\phi_{i,j}^k$ shown in the right panel of Figure 9.

Motherhood and child rearing are captured by the state $k \in \{0, 1, 2, 3\}$. Women start their life childless ($k = 0$). Once they give birth, they are assigned state $k = 1$, which indicates the presence of small children (aged 0-5). After an average of 6 years, mothers of small children transition into $k = 2$, indicating the presence of older children (aged 6-17). Children leave the house after a total of 18 years on average. Once children have left the house, mothers are assigned state $k = 3$.

Figure 9: Motherhood and survival



Consequently, the transition matrix $\pi_k(k^+|k, j, i, 1)$ for state k reads

$$\pi_k(k^+|k, j, i, 1) = \begin{bmatrix} 1 - \phi_{i,j}^k & \phi_{i,j}^k & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{11}{12} & \frac{1}{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{28}$$

Note that women in our model can only give birth to two children once, i.e. $k = 3$ is an absorbing state. Note further that, owing to the limitation to two children, the total fertility rate in our model is 1.40, which is slightly below the 2017 value of 1.57.

B.2 The Structure of the Labor Market

A crucial element for the analysis of any fiscal reform – and pension reforms in particular – is a model that can replicate empirical labor supply patterns across demographic groups and over the life cycle. Labor supply choices are to a large degree determined by labor productivity and preferences. To discipline our choices of the various model parameters that can impact individual labor supply decisions, we proceed as follows: We first estimate average labor productivity profiles and risk processes for men from administrative data of the German pension insurance system. These estimates deliver the general productivity profiles $z(j, s, \eta)$ for workers at different ages j, education levels s and productivity shocks η. We assume that women share the general productivity process with men, but on top they are exposed to a gender wage gape w_{gap} as well as a motherhood wage penalty $w_{pen}(k)$, which we estimate from data by Schrenker and Zucco (2020).

After all these model parameters are set from direct empirical estimates, we calibrate the remaining parameters of the model so as to tackle empirical statistics on labor supply from the German 2017 Microcensus shown in Table 7. The data show quite distinct patterns of labor supply across genders, age groups and family status. Labor supply at both the extensive and the intensive margin is, for example, much smaller for older men and women as well as for women that live in a marriage. Single women, on the other hand, work on a full-time job more often than married women. While of course, in a non-linear model, every parameter causes changes in several variables, we will try to identify the data moments onto which a parameter or a set of parameters has the most direct impact. At the end of this appendix, we provide a detailed table of exogenously set and endogenously calibrated parameters as well as their data sources or calibration targets, respectively.

Table 7: Benchmark: labor supply

	Women				**Men**	
	not empl	mini-job	PT	FT	not empl	FT
	Young: Ages 25-44					
Data	27.86	5.94	26.09	40.11	13.30	86.70
Model	23.26	6.26	24.31	46.16	12.57	87.43
	Older: Ages 45-63					
Data	25.09	8.47	31.95	34.50	19.64	80.36
Model	25.63	7.41	33.93	33.03	19.97	80.03
	Singles: Ages 25-63					
Data	22.98	4.03	21.66	51.33	23.73	76.27
Model	22.69	0.01	17.53	59.77	19.44	80.56
	Couples: Ages 25-63					
Data	27.71	8.73	32.55	31.01	12.73	87.27
Model	25.14	9.77	33.86	31.22	14.52	85.48

Data Source: RDC of the FSO 2021, own calculations.

B.2.1 Labor hours choices

Labor supply ℓ is modeled as discrete choice, see the discussion in Section 3.2. According to the German Microcensus, full-time employees work an average of 40.4 hours per week, whereas part-time employees work 21.3 hours per week. Assuming a maximum time endowment of 100 hours per week (the equivalent to $\ell = 1$) for an individual,[24] we set $\ell_{full} = 0.404$ and $\ell_{part} = 0.213$. For mini jobs,

[24]Out of a total of 168 hours per week this means that 49 are reserved for sleeping and another 19 for eating, hygiene, etc.

we finally set $\ell_{mini} = 0.1$ paying tribute to the fact that those jobs are typically low-hours marginal types of employment.

B.2.2 General Labor Productivity

All individuals of an education level s share a common deterministic age-specific labor productivity profile $\theta_{j,s}$. We estimate these profiles using administrative data from the German public pension insurance (Deutsche Rentenversicherung). Their administrative dataset – the scientific use file of the Versichertenkontenstichprobe 2017 (FDZ-RV, 2017) – covers the insurance accounts of 69,520 individuals actively insured under the public mandatory German pension scheme.[25] The dataset contains detailed information on the monthly history of pension claims earned by each individual as well as the sources these claim were derived from. Pension claims are in general a good indicator for estimating earnings processes, as they are proportional to individual earnings.[26] We restrict the sample to men aged between 25 and 60 who were not working on a mini-job[27] or long-term unemployed or in another way out of the labor force to obtain a sample in which the vast majority of workers works full time. We finally convert the monthly earnings history into annual values.

Using these data, we estimate the following flexible functional form for the education-specific age-productivity profiles:

$$\theta_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3. \quad (29)$$

This form is flexible enough to capture both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < j_R$) life-cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_{M,s}$ onward. The upper part of Table 8 shows the estimation results, the left panel of Figure 3 visualizes the estimated labor productivity profiles as well as their empirical counterpart.

Taking residual earnings, we estimate the education-specific parameters of an AR(1)-process for log-productivity as specified in (6). The results are shown in the lower panel of Table 8. The life-cycle average labor productivity profiles exhibit the usual shape, where earnings rise over time and especially so for college graduates. The processes for labor productivity risk are highly persistent, with a somewhat smaller persistence for high-school workers and a larger persistence for college graduates. The overall unconditional process variance ranges at around 28 to 30 log-points. According to the German tax law the earnings-threshold for mini jobs is 450 Euros per month. Since not every mini-job worker earns the maximum

[25] The German pension scheme covered 38 million actively insured individuals in 2017.

[26] We adjust the data in the case of earnings from so-called midi-jobs, which are subject to a reduced social security contribution at varying rates.

[27] We cover those explicitly and separately in our model.

Table 8: Parameter values of labor productivity profiles and risk

	High School $s = 0$	College $s = 1$
Intercept $b_{0,s}$	-2.0732	-17.2099
Linear age term $b_{1,s}$	0.7833	11.8163
Quadratic age term $b_{2,s}$	-0.0572	-2.6345
Cubic age term $b_{3,s}$	-0.0026	0.1984
Stagnation threshold $j_{M,s}$	∞	44.26
Autocorrelation ρ_s	0.9300	0.9900
Innovation Variance $\sigma^2_{\varepsilon,s}$	0.0372	0.0059
Unconditional Variance $\frac{\hat{\sigma}^2_{\varepsilon,s}}{1-\hat{\rho}^2_s}$	0.2756	0.2983

amount, we assume mini-job earnings of 400 Euros per month that corresponds to 4800 Euros annually or $\bar{y}_{mini} = 0.1297 \times \bar{y}$.

B.2.3 Gender Wage Gap

While we assume that men and women share the same general productivity level $z(j, s, \eta)$, they still face different gross wages in the data. The gray triangles in the right panel of Figure 3 show the gross wage gap between employed men and women over the life cycle as estimated by Schrenker and Zucco (2020). To account for age-specific wage difference between men and women, we assume that women are exposed both to a uniform wage gap w_{gap} that lowers the wages of all women relative to men, and a motherhood wage penalty $w_{pen}(k)$ that is specific to mothers. We therefore pay tribute to recent empirical evidence pointing to the fact that today, the majority of earnings differences between men and women can be attributed to the consequences of having children, see for example Kleven et al. (2019). Using the life-cycle data from Schrenker and Zucco (2020), we estimate $w_{gap} = 0.87$ as well as a motherhood wage penalty that depends on the presence and age of children as

$$w_{pen}(\cdot) = \begin{bmatrix} 1.00 & 0.80 & 0.68 & 0.57 \end{bmatrix}.$$

Note that the *motherhood penalty* estimated in empirical studies typically combines the effects of hours and wage changes into one statistic. In contrast, our estimates of the pure *motherhood wage penalty* solely focus on wage differentials between mothers and non-mothers, which rather are a consequence of missed opportunities for accumulating specific human capital or climbing the career ladder. As such, the motherhood wage penalty rises with the age of children and is the largest when children have left the house. Using these estimates, our model provides a good fit for the evoluation of the gender wage gap over the life cycle, see the right panel of Figure 3. In Section 4.2 we also provide model simulations for the entire motherhood penalty that combines wage and hours differentials.

B.3 Preferences and the Budget Constraint

Utility is additively separable in consumption $c_{j,g}$ and labor supply $\ell_{j,g}$, see again (7). Utility from consumption features constant absolute risk aversion σ, utility from labor a constant but gender-specific Frisch elasticity χ_g. On top of disutility from working, parents have to bear the time costs of caring for children amounting to $\zeta_{k,i,g}$, see the discussion below. Finally, participation in the labor market is costly to individuals. Specifically, when choosing labor hours greater than zero, a worker has to pay the participation utility cost ξ. We assume that ξ is drawn at the household-level – meaning that it is common to married couples – but iid across households and across time and independent of individual labor productivity. We let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2.

Preference Parameters We assign a value of 2.0 to risk aversion σ, a choice quite typical for the heterogeneous agent macroeconomics literature though at the lower end of values that generate an extensive desire for redistribution.[28] The empirical literature has pointed to the fact that Frisch elasticities differ significantly between men and women, see for example Keane (2011). Consistent with this evidence, we chose values of $\chi_m = 0.4$ for men and $\chi_w = 0.875$ for women. After making these data-based choices, we are left with the set $(\beta, \nu_m, \nu_f, \mu_\xi, \sigma_\xi^2)$ of parameters that we need to calibrate. We choose the time discount factor $\beta = 0.98$ so that all capital is entirely absorbed by private savings in the initial equilibrium, and net foreign assets as well as the trade balance are zero. We then jointly calibrate $\nu_m = 75$, $\mu_\xi = 1.85$ and $\sigma_\xi^2 = 2.5$ to match the participation rates of men across demographic groups in Table 7. Finally we set $\nu_f = 15.25$ to achieve an overall good divide between mini-job, part-time and full-time work for women.

Time costs of children Children need to be taken care of, which we model as time costs when they are present in the household ($k = 1, 2$). Single mothers have to bear the entire time cost of children on their own. We hence calibrate the time cost of young children $\zeta_{1,s,w} = 0.6$ and older children $\zeta_{2,s,w} = 0.15$ to match the labor supply patterns of both single women as well as young women in Table 7. Married mother exhibit a somewhat different labor supply pattern over the life cycle, which might be either due to the fact that they live in a two

[28]In this model, σ fulfils two roles as it defines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter typically range between values of 1 and 3, whereas risk aversion can be quite high and well beyond values of 10 when estimated from individual financial choices, see for example Vissing-Jørgensen and Attanasio (2003).

earner household or may result from partners partially sharing the cost of raising children. In order to match the empirical labor supply profiles of married women, too, we assume that the overall time costs of raising children are the same in single and married couple families, but that fathers take a certain (small) share of these costs. This leads us to $\zeta_{1,c,w} = 0.54$ and $\zeta_{1,c,m} = 0.06$ for young children as well as $\zeta_{2,c,w} = 0.11$ and $\zeta_{2,c,m} = 0.04$ for older children.

Labor hours flexibility Finally, we pay tribute to the fact that many mothers continue to work part-time, even when their children have already left the household. Our model would not be able to adequately replicate this pattern without restrictions on mother's flexibility in choosing labor hours. The state $h \in \{\ell_{part}, \ell_{full}\}$ refers to a woman's choice set for labor hours. The transition matrix $\pi_h(h^+|h, g, \ell)$ governs the transition between these states and is conditional on the current labor hours choice ℓ. For women who work full time, we assume that the transition matrix is the identity matrix, meaning that they will not be at the risk of facing labor hours restrictions in the next period. Women who do not work full-time transition from $h = \ell_{full}$ into the state $h = \ell_{part}$ with a likelihood of 0.9. Once in this state, they come back to $h = \ell_{full}$ with an annual probability of 0.2. The average duration of a period of labor hours inflexibility is therefore 5 years.

B.4 Technology

We choose a depreciation rate of $\delta = 0.07$, which leads to a realistic investment to output ratio of 21%, see German Statistical Office (2020). We set the capital share in production at $\alpha = 0.3$ to obtain a capital-to-output ratio of three and normalize the technology level Ω such that the wage rate per efficiency unit of labor w is equal to 1. Finally, we assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

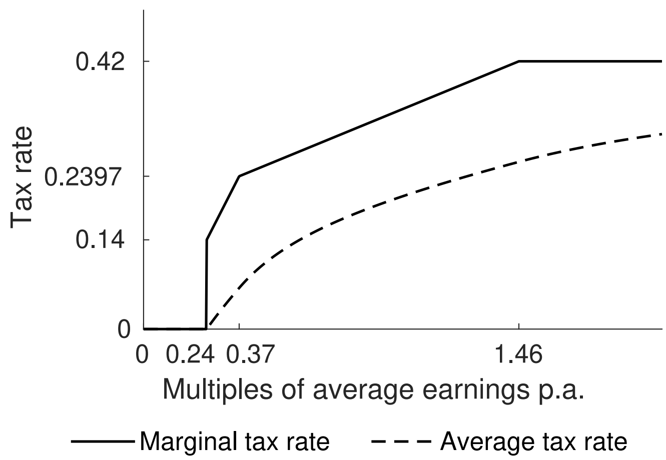
B.5 The Public Pension System

The pension system runs on a pay-as-you-go basis. This means that in equilibrium, total annual pension contributions need to be equal to the total amount of annual pension payments. We fix the pension contribution rate at its statutory rate of $\tau_p = 0.187$ in 2017. The accrual rate for mini-job earnings is $\varrho = 0.80$. These choices result in a gross pension replacement of $\kappa = 0.41$, which is similar to the German gross replacement rate for the mean earner as reported by OECD (2021).

B.6 The Tax System and Government Expenditure

The government raises proportional taxes on consumption and progressive taxes on labor earnings to cover government expenditures. We employ the 2017 statutory German progressive income tax code as depicted in Figure 10 . Recall that couples enjoy a tax advantage in the form of income splitting, see (16). We set the proportional consumption tax rate at $\tau_c = 0.17$ to balance the fiscal budget. This untargeted tax rate is in line with the German VAT tax. Although consumption goods are generally taxed at a rate of 19%, many goods (such as food, books and newspapers) are taxed at a lower rate. We fix (wasteful) government consumption at 19% of GDP in the benchmark economy, see German Statistical Office (2020).

Figure 10: Labor tax schedule



The government provides two direct transfer programs to support families with children. In 2017, parents received a child benefit of EUR 192 per child and month. Moreover, we let the government pay additional tax financed child support payments to single mothers, which mimic both alimony payments but also subsistence transfers in the real world. We set these monthly child support payments to EUR 576 per child.[29] With average labor earnings of 37,000 Euros, the child transfer function consequently reads

$$t(k,i) = \begin{cases} 0.1245 \times \bar{y} & \text{if } k \in \{1,2\} \text{ and } i = c \\ 0.4980 \times \bar{y} & \text{if } k \in \{1,2\} \text{ and } i = s \\ 0 & \text{otherwise.} \end{cases}$$

[29]In Germany, alimony payments depend on individual income and the age of the child. The so-called "Duesseldorfer Table" specifies the exact amount. For monthly net earnings between 1,500-5,100 Euro, monthly payments amount 360-736 Euro in 2017.

In addition to direct transfers, mothers are compensated for foregone pension contributions while raising children. For each child, they are credited pension entitlements equivalent to that of an average earner for three years. In order to account for this in the model, we set the child pension credit to $t_{child}(k = 1) = 1$ and to zero otherwise, as the two children borne to a mother remain small ($k = 1$) for an average of six years.

B.7 Parameter values - overview

Table 9 summarizes calibrated parameters as well as their targets. Table 10 summarizes all model parameters that were taken directly from the literature or estimated directly from data.

Table 9: Summary of endogenous model parameters

	Value	Source
Depreciation rate δ	0.070	Investment/Output: 21.0
Technology level Ω	0.923	wage per efficiency unit $w = 1$
Discount factor β	0.98	Closed economy: NFA = 0.00
Consumption tax rate τ_c	0.17	Government Budget balance
Replacement rate κ	0.41	Pension Budget balance
To target labor supply		Data from Table 7
Disutility labor ν_m	70.00	
Disutility labor ν_f	22.00	
Disutility empl. mean μ_ξ	1.65	
Disutility empl. var σ_ξ^2	2.50	
Time costs		
- young kids/single mothers $\zeta_{1,c,f}$	1.50	
- older kids/single mothers $\zeta_{2,c,f}$	0.25	
- young kids/married mothers $\zeta_{1,c,f}$	1.20	
- older kids/married mothers $\zeta_{2,c,f}$	0.1875	
- young kids/married fathers $\zeta_{1,c,m}$	0.30	
- older kids/married fathers $\zeta_{2,c,m}$	0.0625	

Table 10: Summary of exogenous or estimated model parameters

	Value	Source
Demographics		
Fraction of couples ϕ_c	0.677	Microcensus 2017
Fraction of women ϕ_g	0.492	Microcensus 2017
Fraction college educated ϕ_s^m	0.330	Microcensus 2017
Fraction college educated ϕ_s^f	0.277	Microcensus 2017
Assortative mating:		Microcensus 2017
$\phi_s^c(0,0)$	0.5740	Microcensus 2017
$\phi_s^c(1,0)$	0.1492	Microcensus 2017
$\phi_s^c(0,1)$	0.0958	Microcensus 2017
$\phi_s^c(1,1)$	0.1810	Microcensus 2017
Fertility women:		Microcensus 2017
ϕ_k^s	0.475	Microcensus 2017
ϕ_k^c	0.800	Microcensus 2017
Max. age J	99	Assumption
Population growth rate n	0.000	Population register data
Retirement age j_r	64	DRV Bund (2019)
Others		
Child benefits t_{cb}	$0.1245\bar{y}$	German Statistical Office (2020)
Child benefits t_{cs}	$3 \times t_{cb}$	Düsseldorfer Tabelle
Mother benefits e_k	1	German Statistical Office (2020)
Returns to scale $\upsilon(j,k,i)$		new OECD equiv. scale
Pension contribution rate τ_p	0.187	DRV Bund (2020)
Pension share mini-jobs ϱ	0.80	DRV Bund (2020)
International interest rate $\bar{r}$	0.030	
Capital share in production α	0.300	
Intert. elast. of substitution σ	0.500	Heathcote et al. (2014)
Frisch elast. of labor supply χ_m	0.400	Keane (2011)
Frisch elast. of labor supply χ_f	0.750	Keane (2011)
Gender gap w_g	Figure 3	Schrenker and Zucco (2020)
Survival probabilities $\bar{\psi}_{j,g}$	Figure 9	HMD (2020)

C Datawork

The productivity profiles in this paper are based on administrative data from the German Pension Insurance. In particular we use the 2017 wave of the scientific use-file of the Versichertenkontenstichprobe that contains monthly earnings data of 69,520 insured individuals. This is about 0.18% of the actively insured population.[30] We restrict our attention to the male sample population aged between 25 and 60 of which we have information on the education level. Our measure of monthly labor earnings comprises income from regular work, marginal employment and short-term unemployment (up to one year). We count all other source of pension accumulation (like times of care for children or sickness) as zero earnings months. We sum up monthly earnings observations to construct an annual earnings measure for each individual. This appendix explains the data selection and estimation process in detail.

C.1 The Administrative Dataset

We use the same dataset and the same estimation strategy as Kindermann and Pueschel (2023). However, our approach differs with respect to low earnings individuals. Kindermann and Pueschel (2023) define a low-earnings group (observations on annual earnings ($y < 0.23\bar{y}$) that is excluded from the sample. We, instead, simply exclude mini-job worker from the sample.

The data set consists of two parts: One provides demographic characteristics such as age, gender and education for the year 2017. The other one records the entire history of an individual's accumulated pension claims and employment status on a monthly basis. The sample covers worker who were born between 1950 and 1987 and who were not permanently retired in 2017. The historical record starts in the year an individual turns 14 and ends when she turns 65. Hence, the maximum length of an employment history is 624 month. Overall, the data set includes more than 28 million worker-month observations for the years 1964 to 2017. As the sample ends in December 2017, individuals who were born in 1953 or later have shorter histories (e.g. 612 month for the 1953 cohort). Those who have never been employed are not represented, as they never were registered with the insurance.

C.1.1 Earnings measurement

Earnings y_{isjt} of an individual i of education s and age j at time t are subject to social security contribution. There is a contribution threshold $y_{max,t}$ and any earnings beyond that value are non-contributory. Contributory earnings hence amount to $\min(y_{isjt},\ y_{max,t})$. They are converted into pension claims y^p_{isjt} by diving them through average earnings $\bar{y}_t$. We account for the fact that pension claims

[30] The German pension scheme covered of 38 million actively insured individuals in 2017.

from so-called mini-and midi jobs[31] are subject to a reduced pension contribution rate. Both, the contribution threshold $y_{max,t}$ and average earnings $\bar{y}_t$ are adjusted annually to account for wage growth. The contribution threshold $y_{max,t}$ currently amounts to about twice the average earnings $\bar{y}_t$.[32]

For our analysis, it is most convenient to use pension claims y^p_{isjt} as an earnings measure, as they are stationary over time. In particular, we define

$$y^p_{isjt} = \frac{\min\left(y_{isjt},\ y_{max,t}\right)}{\bar{y}_t}. \tag{30}$$

Although the monthly records start in 1964, we only consider observations for the years 2000 to 2016. This has certain advantages: First, our estimates are based on recent data; second, we avoid structural breaks arising from German reunification and policy-changes in the 1990s and third, different age cohorts are represented in the sample at similar shares in each year (early sample years cover only young individuals). The data-selection process is summarized in Table 11.

Table 11: Data Selection

	Individuals	Observations
Initial data set (1975 - 2017)	69,520	28,166,952
Initial data set (2000 - 2016)	69,520	14,139,972
- Women	−36,634	−7,451,736
- Ages < 25		−1,014,120
- Ages > 60		−152,976
	32,886	5,521,140
- Ind. that receive pensions	−3,606	−605,208
	29,280	4,915,932
- Ind. with unknown education	−13,677	−2,346,840
	15,603	2,569,092
Annualized data (2000 - 2016)	15,603	214,091
No contributory earnings	−391	−25,197
Final data set	**15,212**	**188,894**
Non-college education	11,800	149,757
College education	3,412	39,137

We restrict the sample such that it targets workers who are attached to the labor market. We therefore limit our attention to men aged between 25 and 60 who are

[31] In a mini-job, an individual can earn a maximum of EUR 450. Midi-jobs cover earnings from 451 to 850 Euros.

[32] See Section 11 in DRV Bund (2020) for a full history of reference values.

likely to already have finished education and military service and are not in the process of retiring. We drop all individuals who already received pensions such as disability pensions or early-retirement pensions.

We divide the sample into two educational groups. We adapt the scheme to the International Standard Classification of Education of the UNESCO (ISCED 2011) to allow for international comparison. A person is defined to be college-educated[33] if she is classified ISCED 6 (Bachelor's or equivalent level) or above, excluding ISCED 65 (trade and technical schools, including master craftsman training). She is non-college-educated[34] if she is classified ISCED 5 and below or ISCED 65. We drop those with unknown education status.

For estimating earnings profiles we use all pension claims y^p_{isjt} that stem from regular-employment or unemployment benefits (short-term, max. 12 month) according to the variable `SES`. Since individuals are productive when searching for a new job, we consider short-term unemployment as an employment type. Since mini-job worker in our model are payed a fixed salary, we exclude that group from the data set. Figure 11 shows the distribution of earnings. Obviously, the data are right-censored at $y_{max,t}$.

Figure 11: Histogram of pension claim distribution

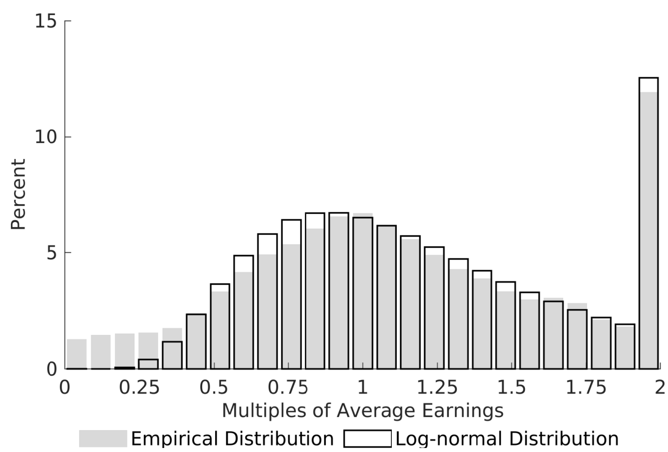


Table 12 shows the distribution of employment states across monthly observations. About 13 percent of all observations are on months with no contributory earnings (12.9 percent). Such observations emerge when individuals become self-employed or civil servants, when they take care leave, face a longer spell of unemployment or just decide to drop out of the workforce. About 1.7 percent are on months with earnings from a mini-job, which we code as no contributory earnings as well. In the model, mini-job worker usually receive a flat salary and are hence independent of the regular earnings process. Hence, we exclude mini-job earnings from the

[33]corresponds to KldB 2010 4-6

[34]corresponds to KldB 2010 1-3

sample. Only the least productive mini-job worker a payed in accordance with their productivity, see Section 3.2. We code non-contributory months as periods of zero earnings.

Table 12: Distribution of Employment States (across monthly observations)

Employment Status	Observations	Percent
Regular employment	2,139,302	83.27
Unemployment (short-term)	55,138	2.15
No contributory earnings	374,652	14.58
of which mini-job earnings	44,113	1.72
Total	2,569,092	100.00

To make the data comparable with our simulation model, we have to change the time-dimension of the panel from monthly to annual. We do so by computing the sum of acquired pensions claims for each calendar year. Finally, we drop all sample individuals who had no contributory earnings at all in the period from 2000 to 2016. We exclude observations with no contributory earnings in an entire calendar year, see Table 11. Our final data set is an unbalanced annual panel for the years 2000 to 2016 with 15,212 individuals – of which 22.4 percent are college-educated – and a total of 188,894 observations.

C.2 Earnings estimates

In the following, we describe the estimation process for the life cycle earnings profiles and labor earnings risk of regular workers in detail.

C.2.1 Identifying the top censoring threshold

Our starting point is the data set of regular workers with 188,894 observations as summarized in Table 11. First, we need to identify the top censoring threshold. Although the German public pension insurance provides an official contribution ceiling $\tilde{y}_{max,t}$ for contributory earnings in every year, see DRV Bund (2020), we cannot take this value directly. The reason is that the ceiling is applied on a monthly basis while we are working with annual data. Hence, an observation could be subject to censoring, although the observed annual earnings y^p_{isjt} are below the official cut-off value. This is the case if the contribution threshold is reached in some months of the year, but not in others (for instance because of salary changes). In addition, we observe a few outliers where annual pension claims y^p_{isjt} are beyond the corresponding official threshold, which might be due to value adjustments.

To overcome these problems, we use the following strategy to identify a threshold $y_{max,t}$ for every year that allows us to capture most observations that have been top-coded at least in one month:

1. First, we find the value of pension claims $mode_{y,t}$ at the upper end of the distribution where most of the observations pile up and compare it to the official threshold $\frac{\tilde{y}_{max,t}}{\bar{y}_t}$. $mode_{y,t}$ typically is in the order of 0.0002 smaller than $\frac{\tilde{y}_{max,t}}{\bar{y}_t}$, which corresponds to about 7 Euros in 2016 compared to an average income of 36,000 Euros.

2. We then define our censoring threshold as

$$\frac{y_{max,t}}{\bar{y}_t} = mode_{y,t} - 0.0003.$$

   This guarantees that (i) $y_{max,t}$ is always smaller than $\tilde{y}_{max,t}$ and (ii) as little information as possible is cut off.

3. Next, we identify outliers as observations with

$$y^p_{isjt} > 1.05 \times \frac{y_{max,t}}{\bar{y}_t},$$

   that is those that exceed the contribution ceiling by more than 5 percent. These outliers are treated as observations with no contributory earnings and therefore deleted from the data set (285 observations).

4. Finally, we recalculate pension claims for all individuals that exceed the contribution ceiling by less than the outlier threshold. Specifically, we set

$$y^p_{isjt} = \frac{y_{max,t}}{\bar{y}_t} \text{ for all } i \text{ with } y^p_{isjt} > \frac{y_{max,t}}{\bar{y}_t}.$$

   We therefore have to modify 16,597 observations.

After these steps, the data is subject to a sharp annual censoring threshold $y_{max,t}$, which is required for the estimation. Table 13 shows the exact values of $\tilde{y}_{max,t}$, $y_{max,t}$, and the share of observation at both thresholds for each year. About 7 to 11 percent of the annual observations are on the threshold value $y_{max,t}$.

C.2.2 Statistical Model

We describe the earnings dynamics of the normal earner sample by a standard AR(1) process in logs. We therefore split the normal labor earnings sample according to an individuals' education level $s \in \{0, 1\}$. $s = 0$ summarizes all individuals with high school education, while $s = 1$ indicates the college educated workforce. For each education group, we estimate the statistical model

$$\log(y_{isjt}) = \kappa_{t,s} + \theta_{j,s} + \eta_{isjt} \quad \text{with} \quad \eta_{isjt} = \rho_s \eta_{isj-1,t-1} + \varepsilon_{isjt}, \qquad (31)$$

Table 13: Identification of $y^*_{max,t}$

Year t	$\tilde{y}_{max,t}$	% at $\tilde{y}_{max,t}$	$y_{max,t}$	% at $y_{max,t}$	Observations n
2000	1.9021	0.8821	1.9017	8.7230	7,142
2001	1.8908	8.1519	1.8905	9.2273	7,532
2002	1.8864	1.1574	1.8858	9.6577	8,035
2003	2.1149	0.2833	2.1143	6.9043	8,473
2004	2.1266	0.5959	2.1261	7.2633	8,894
2005	2.1368	7.1505	2.1365	7.3322	9,356
2006	2.1360	6.9863	2.1358	7.1164	9,991
2007	2.1034	0.9088	2.1029	8.1694	10,674
2008	2.0767	0.9809	2.0763	8.7929	11,316
2009	2.1242	0.3949	2.1239	8.0743	11,902
2010	2.1192	8.1967	2.1191	8.2286	12,566
2011	2.0561	0.6428	2.0556	9.2339	13,223
2012	2.0362	9.1126	2.0361	9.2573	13,827
2013	2.0678	9.3192	2.0675	9.7438	13,896
2014	2.0687	0.6930	2.0683	9.9228	13,998
2015	2.0530	10.3960	2.0528	10.4461	14,034
2016	2.0560	0.7481	2.0553	11.3146	14,035
					188,894

* Values for $\tilde{y}_{max,t}$ and $y_{max,t}$ are expressed relative to average earnings $\bar{y}_t$.

for labor earnings y_{isjt} of an individual i with education s at age j in year t. $\kappa_{t,s}$ is a year fixed effect that controls for earnings changes along the business cycle. $\theta_{j,s}$ is an age fixed effect that informs us about the age-earnings relationship. The noise term ε_{isjt} is assumed to follow a normal distribution with mean 0. Furthermore, we let the stochastic process start from its long-run variance σ_s^2. This means that

$$\varepsilon_{isjt} \sim N(0, \sigma^2_{\varepsilon,s}) \quad \text{and} \quad \eta_{is20t} \sim N\left(0, \sigma_s^2\right) \text{ with } \sigma_s^2 = \frac{\sigma^2_{\varepsilon,s}}{1-\rho_s^2}.$$

We use a generalized method of moments estimator to determine the parameters of this model. We thereby control for the fact that the data are top-coded at the threshold $y_{max,t}$.

C.2.3 Moment Conditions and Estimation

To estimate the statistical model in (31) with our data, we have to determine a total of 110 parameters:

1. 34 year fixed effects $\kappa_{t,s}$ for the years 2000 to 2016 and the education levels $s \in \{0, 1\}$;

2. 72 age fixed effects $\theta_{j,s}$ for the ages 25 to 60 for each education level s;
3. the two unconditional variances σ_s^2;
4. the two autocorrelation parameters ρ_s.

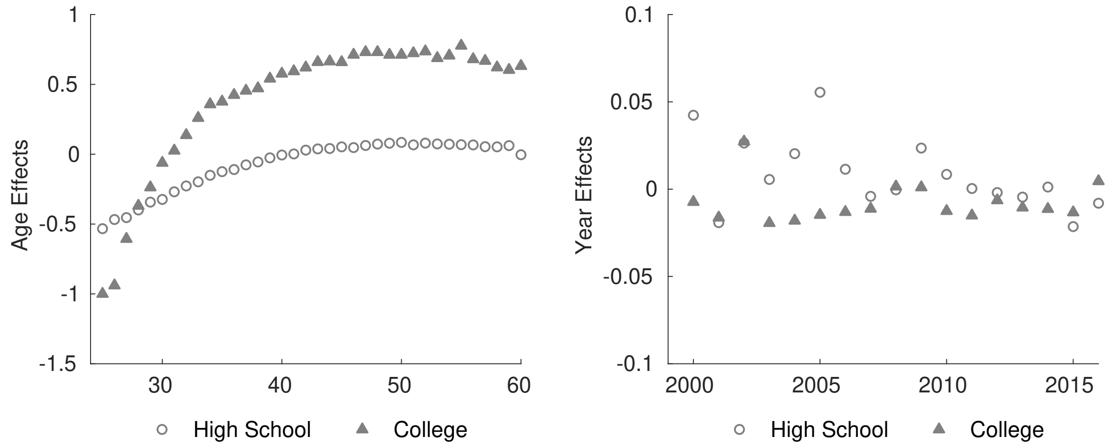
In order to estimate these parameters, we use the labor earnings data y^p_{isjt} to calculate the empirical moments that correspond to the means E_{sjt}, censoring shares P_{sjt}, variances Var_{sjt} and covariances Cov_{sjt} discussed above for each education level s, age j and year t. We exclude moments when the number of individuals in the corresponding education-age-year bin is smaller than 30, or when the empirical standard error of the moment is equal to zero. This gives us the following moments:

- **sample means:** we estimate 974 means $\hat{\mu}_{sjt}$ of $\log(y^p_{isjt})$ including the censored observations $y_{isjt} = y_{max,t}$ and the corresponding standard errors $\frac{\hat{\sigma}_{sjt}}{\sqrt{n_{sjt}}}$;
- **share of observations at threshold $y_{max,t}$:** we compute 930 shares $\widehat{shr}_{sjt}$ of the observations that sit exactly on the threshold $y_{max,t}$ and the corresponding standard errors $\sqrt{\frac{shr_{sjt}(1-shr_{sjt})}{n_{sjt}}}$;
- **sample variances:** we estimate 943 variances $\hat{\sigma}^2_{sjt}$ of $\log(y^p_{isjt})$ excluding the censored observations as well as the corresponding standard errors of the variance $\hat{\sigma}^2_{sjt}\frac{\sqrt{2}}{n_{sjt}-1}$;
- **sample covariances:** we compute 877 covariances $\hat{\sigma}_{sjt,t+1}$ of $\log(y_{isjt})$ excluding the censored observations as well as the corresponding standard errors of the covariance $\sqrt{\frac{(\hat{\sigma}_{sjt,t+1})^2+\hat{\sigma}^2_{sjt}\hat{\sigma}^2_{sj+1,t+1}}{n_{sjt}-1}}$.

We use these 3724 empirical moments to calculate a residual sum of squares measure. We use a diagonal weighting matrix that has the inverse of the squared standard errors of the empirical moments on the diagonal. To minimize the residual sum of squares and account for multiple local minima, we use the method of simulated annealing, see Du and Swamy (2016). We estimate parameters separately for each education level s.

The results of this estimation process are quite standard in the sense that the estimates exhibit typical life cycle labor earnings profiles, a significant college wage premium as well as a high auto-correlation of earnings, see Figure 12. We will use these estimates as prime inputs into the calibration of our quantitative model. Yet, as the statistical model describes labor earnings and not labor productivity, we can not use the estimated parameters as direct inputs. The left panel of Figure 12 visualizes the point estimates of the age fixed effects by education level. Up to the age of 45, earnings steeply increase for both education groups, especially so

Figure 12: Age fixed-effects and year fixed-effects



for the college educated. Afterwards, they stagnate or decline slightly for the rest of an individual's working life. This shape of life cycle earnings is quite common in the empirical literature and has been found for other countries as well, see for example Heckman et al. (1998) or Casanova (2013). The college-wage premium implied by these profiles is equal to 60 percent, which is in line with empirical findings (OECD, 2016). The right panel of the figure shows the year fixed effects. These are generally small relative to the age effects and exhibit some cyclical dynamics. Table 8 summarizes the estimation results for the residual earnings process.

D Further Simulation Results

Table 14: Individuals who retire with $e_{j_r} < b\bar{y}$ in the benchmark economy

	No kids		**Kids**	
	n. College	College	n. College	College
Singles				
Women	1.38	0.76	15.19	7.94
Men	0.84	0.61	-	-
Couples				
Women	16.10	11.40	22.81	17.62
Men	2.64	2.12	1.87	1.46
Married Women				
Man HS	13.87	6.54	20.70	11.25
Man Col	24.68	13.98	30.92	20.99
Married Man				
Women HS	1.97	1.06	1.55	0.80
Women Col	6.66	2.99	3.74	2.01

Figure 13: Changes in employment rates men