Women's Labor Supply Incentives and Old-Age Income Redistribution

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Abstract

The risk of ending up poor in old age is shaped at young ages and it is concentrated among women. To counteract old-age poverty, many countries redistribute income through the pension system. They often do so based on an individual's lifetime earnings, like US Social Security. In this paper, we argue that a pension system that uses annual instead of lifetime earnings as basis for old-age income redistribution can lead to much better labor market outcomes and a superior old-age income distribution. We show both theoretically and quantitatively that such a system comes with broad employment incentives, especially for individuals prone to old-age poverty risk. As such, it addresses the causes of old-age poverty and not only its consequences. Our quantitative simulation model includes rich demographics and a detailed model of female labor supply. We account for gender, family status, children, and labor supply choices at the intensive and extensive margin. While lifetime-earnings-based redistribution causes substantial long-run welfare losses, annual-earnings-based redistribution increases long-run welfare, particularly for women.

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1 Introduction

The risk of ending up in old-age poverty is not exclusively exogenous. Instead, it is shaped by active choices that individuals have made throughout their working life. And these choices, in turn, often depend on family and children. It is, hence, not surprising that "old-age poverty has a woman's face" in both the developed and the developing world (United Nations, 2022). Women – and especially single mothers – are identified as one of the major risk groups for old-age poverty already today. Yet, they are also projected to suffer disproportionately from likely increases in old-age poverty over the next decades, see Haan et al. (2017). Many countries try to counteract this problem by redistributing income through the pension system. This income redistribution is usually based on an individual's lifetime earnings. In this paper, we argue that such systems miss out on an important opportunity. They only address the consequences of old-age poverty risk but not its causes, specifically an unsteady employment history.

We study the consequences of fiscal redistribution through the pension system for individual and aggregate labor supply, for the distribution of old-age income and for welfare. To this end, we develop and parameterize a detailed quantitative simulation model that accounts for a wide range of demographics and labor market characteristics. We pay particular attention to the trade-offs that shape life-cycle labor supply and old-age income of women both at the extensive and the intensive margin. Our model allows us to compare two different types of redistributive pension systems: First, a *lifetime-earnings-based progressive pension* that, similar to US Social Security, conditions redistribution on the lifetime earnings of an individual. Second, an *annual-earnings-based progressive pension* that, similar to a tax system, uses individual earnings of a given year to determine the degree of income redistribution in old age.

The two types of progressive pension systems are remarkably different in terms of their effects on labor supply, on the old-age income distribution and on long-run welfare. The major cause of these differences are the labor supply incentives embedded in the two systems. We show both theoretically and quantitatively that a lifetime-earnings-based progressive pension discourages labor supply for a broad share of the population. These negative distortions are at work both at the extensive and the intensive margin. Negative distortions especially lead poorer individuals to take home fewer earnings during working life. More importantly, however, they also depress the distribution of pension claims, as a lack of earnings years is only partly compensated by pension progressivity. The overall result is a lower level of labor force participation and a less favorable distribution of old-age pension income. It is, hence, not surprising that such a system comes at substantial long-run welfare losses for almost all individuals.

An annual earnings based progressive pension instead has the potential to unleash positive labor supply incentives for a broad set of individuals. We show that it is able to encourage labor force participation for all workers who earn less than the economy's average earnings. In addition, it pushes women from marginal towards regular employment. As such, an annual earnings based progressive pension comes at a more favorable distribution of primary income and also of old-age pension income. The positive effect on labor force participation counteracts labor supply distortions at the intensive margin. An annual earnings based progressive pension consequently tackles one of the most important causes of old-age poverty risk: an unstable career and a lack of earnings years. In doing so, it raises long-run welfare and especially so for women.

The starting point for our analysis is an economy with a purely proportional pension system, meaning a system in which pension payments rise one to one in individual earnings. We introduce redistributive pensions into this economy by modifying the pension calculation formula with a progressive function $f(\cdot)$. This progressive function is oriented towards the calculation of the primary insurance amount (PIA) in US Social Security. In particular, it features a steep subsidy region for low income earners, in which workers accumulate disproportionately large pension payments. After a bend point, the calculation formula flattens considerably, leading to a much lower pension replacement rate for medium and high income earners.[1] Mechanically, the difference between a lifetime-earnings-based and an annual-earnings-based progressive pension lies in the point in time at which the progressive formula is applied. In a lifetime-earnings-based system, we calculate an individual's lifetime average earnings and then derive the individual pension payment from a progressive transformation of these lifetime earnings. Put simply, such a system calculates pension benefits as $p = f(\sum_j y_j)$. An annual-earnings-based pension, on the other hand, already applies the progressive formula to annual earnings and then calculates pensions from a lifetime average of these transformed earnings. Sticking with our simple notation, this system calculates pension benefits from $p = \sum_j f(y_j)$.

We compare the economic differences between lifetime- and annual-earnings-based redistribution using a quantitative macroeconomic model with heterogeneous individuals and overlapping generations. In order to provide an adequate picture of the heterogeneity of pensioners and their risk of poverty in old age, we account for a wide range of demographics and labor market characteristics. We consider individuals of different genders who can either live as single households or in a marriage. Both single and married women may give birth to children, which is costly in terms of time and resources. We distinguish individuals according to their education and account for assortative mating in the marriage market. Furthermore, we allow for differences in labor productivity that arise from persistent productivity shocks, gender discrimination and a motherhood wage penalty. We provide a detailed model of individual labor supply decisions at the extensive and, for women, also at the intensive margin. Couple households have to solve a two-earner problem, where we allow for specialization especially under the presence of young children. Women can choose to work on full-time, part-time or marginal

[1] Figure 1 shows the exact formula we use, and we discuss its properties more thoroughly in Section 2.

working contracts.

We calibrate this model to the German economy, which currently features a proportional pension system. Our calibrated simulation model is able to match the empirical labor supply profiles of different population subgroups, the motherhood penalty as well as the distribution of pension claims of the German economy. Germany is the largest economy in Europe and it faces a demographic transition that is representative of many Western economies. It exhibits levels of female labor force participation and gender-specific risks of old-age poverty that are comparable to many Western countries. Old-age poverty risk rates in Germany are increasing, especially so for single women. More than 50 % of them are projected to be at risk of poverty in old age in 2031-36, see Haan et al. (2017). This has led to a debate on whether and how to adapt the pension system to this situation. The 2023 annual report of the German Council of Economic Experts, for instance, promotes a pension reform that would transition from the current proportional to a progressive pension system. A recent study by Breunig et al. (2022) shows that a majority of Germans rejects the strict proportionality between lifetime earnings and pension benefits and would support more redistribution through the pension system.

When we introduce progressive pensions into the German economy, our simulation results reveal significant differences between a lifetime- and an annual-earnings-based progressive pension. Relative to a proportional pension, both systems distort labor supply along the intensive margin. Women reduce their working hours by about 0.3 hours per week in both reform exercises. This effect is standard and relates to the equity-efficiency trade-off embedded in all redistributive fiscal systems. However, we find notable differences at the extensive margin. Aggregate employment increases by 0.1 percentage points in the annual-earnings-based system, but it drops by 1.1 percentage points in the lifetime-earnings-based system. The annual-earnings-based system stimulates employment of individuals who are on the margin of dropping out of the labor force. Those are predominantly mothers of young children and elderly workers who wish to retire early either because they experience a negative productivity shock or because they already had a successful career. The rise of aggregate employment partly counteracts the distortion of intensive margin labor supply. Consequently, long-run macroeconomic performance is much better under annual earnings redistribution than under lifetime earnings redistribution. Both progressive pensions generally reduce old-age income inequality. However, the annual-earnings-based system creates a superior distribution of pension claims with a smaller left and a fatter right tail. In particular, the share of married women without any own pension claims drops substantially. Finally, we show that a pension system with annual income redistribution leads to long-run welfare gains, while lifetime earnings redistribution causes welfare losses. The difference in welfare effects between the two systems amounts to 0.9 percent measured in consumption equivalent variation.

Relation to the literature Should fiscal redistribution be based on annual or lifetime earnings? This is a fundamental question in public finance and macroeconomics. Back in 1939, Vickrey initiated the discussion by advocating for the averaging of incomes over several years for tax purposes. More recently, his ideas were supported by Haan et al. (2019) or Kapička (2020), for example, who point to the economic gains of a lifetime- or a history-based tax system. Yet, the practical implementation of such systems typically suffers from a number of obstacles. These range from a potential lack of data on the complete individual income history, over the high degree of complexity, up to legal restrictions on age discrimination. The pension system, however, is a natural candidate for lifetime-earnings-based redistribution. This paper analyzes the benefits and costs of lifetime earnings redistribution in the pension system.

More generally we add to a literature on the welfare consequences of changing the redistributive properties of social security systems in heterogeneous-agent life cycle models, among them Huggett and Ventura (1999), Fehr et al. (2013), Nishiyama and Smetters (2008), O'Dea (2018), or Jones and Li (2022). Relative to these studies, we are among the first to study extensive margin effects of social security reforms. The only other study we know so far is Gustafsson (2023) who examines the introduction of a purely Beveridgean pension system. Our paper also connects to the literature on extensive margin labor supply responses and the role for the fiscal tax and redistribution system. Saez (2002) was among the first to show that, when labor supply responses are concentrated along the extensive margin, an optimal labor tax policy explicitly subsidizes employment in a similar way as the Earned Income Tax Credit in the US. A series of studies has quantified the EITC's impact on labor supply, savings, insurance and welfare, including Chan (2013), Athreya et al. (2010), and Ortigueira and Siassi (2022). In contrast, we apply an EITC-style mechanism to the pension system.

The empirical literature has validated that pension reforms can trigger individual labor supply reactions, see for example Coile (2015), Blundell et al. (2016) and Liebman et al. (2009). A study by French et al. (2021) exploits a 1999 pension reform in Poland and confirms that labor supply incentives embedded in pension reforms trigger reactions even about 15 years prior to retirement entry.

Finally, our paper relates to a literature that uses large scale quantitative simulation models with very detailed heterogeneity on the household level. These studies analyze the impact of public policies on individuals of different gender or family type. Examples include Guner et al. (2021), Kaygusuz (2015) or Kurnaz (2021). Amongst them, Kaygusuz (2015) is the closest to our study. He investigates changes in redistributive features of US Social security. Since he abstracts from any sort of earnings risk, he can not quantify any insurance effects.

The remainder of our paper is structured as follows: In Section 2, we analytically investigate the incentive effects embedded in progressive pension systems. In Section 3, we present our full quantitative simulation model and its calibration. Section 4 discusses simulation results and the last section concludes.

2 Building Intuition

We start our discussion with an analytical description of the employment incentives embedded in different redistributive pension systems. We therefore study a stylized version of our quantitative simulation model. Let's consider an individual who starts her working life at age 20 and lives until age J. We denote by j the individual's current age and by j_r her retirement age. In each year of her working life, the individual can decide to work and therefore earn some gross income $y_j = w_j$ related to her current wage w_j. If she decides not to work, her income is $y_j = 0$. For simplicity, we abstract from survival risk and assume an interest rate of $r = 0$. Both assumptions will be relaxed in the full quantitative model. All derivations can be found in Appendix A.

Taxes and benefits of the pension system In each working year, the individual has to pay a contribution at rate τ_p to the pension system. In reward for this contribution, she collects claims to the pension system that define her pension payments in old age. The pension benefit is based on a record of the individual's earnings y_j. In its most general form, we can write the individual's pension as

$$p = \kappa \times F(y_{20}, y_{21}, \dots, y_{j_r-1}),$$

with some function $F(\cdot)$. κ denotes the general replacement rate of the system.

When making the decision to work in a given year j, the individual has to trade off the disutility from working against the monetary benefits. The latter can be summarized in the effective net return to working. In the context of this simple model, the net return to working is

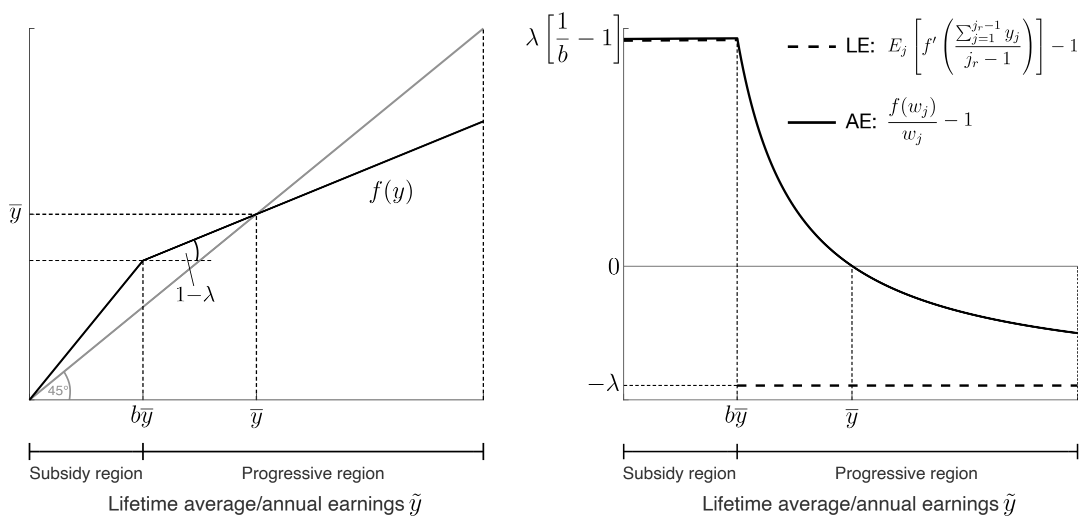
$$\begin{aligned} w_{j,net} = (1-\tau_p)w_j + \kappa \times [J - j_r + 1] \\ \times \underbrace{[F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})]}_{\text{change in earnings record when working at age } j}. \end{aligned}$$

It is determined by two terms: The first term $(1-\tau_p)w_j$ denotes the net annual salary. The second term indicates how the individual's pension payments increase upon working for another year. It is composed of the pension replacement rate κ, the length of the pension payment period $[J - j_r + 1]$, and the change in the earnings record that defines the individual pension payment.

Progressive pension calculation In the following, we want to contrast the employment incentives embedded in different progressive pension systems. To this end, we draw on a simple progressive pension formula $f(\cdot)$ which is inspired by the calculation formula for the primary insurance amount (PIA) in US Social Security. The left panel of Figure 1 shows this formula (black line) and compares it to a proportional function (gray line). As discussed in the introduction, the formula takes either *lifetime average earnings* or *annual earnings* as an argument $\tilde{y}$. It then increases pension benefits for individuals with earnings smaller than the economy's average earnings $\bar{y}$, and it decreases pension payments for the earnings rich. The pension of the average earner remains unchanged relative to a proportional system (grey line). What is more, the formula features a subsidy region in the low earnings segment, in which individuals accumulate disproportionately high pension claims for their earnings $\tilde{y}$. After a bend point $b\bar{y}$, however, additional earnings increase the individual pension benefit only with a factor $1-\lambda$. The parameter λ is a measure for the degree of pension progressivity with $0 \leq \lambda \leq 1$.[2] In analytical terms, the function reads

$$f(\tilde{y}) = \begin{cases} \left[\frac{\lambda}{b} + (1-\lambda)\right]\tilde{y} & \text{in the subsidy region } \tilde{y} < b\bar{y} \text{ and} \\ \lambda\bar{y} + (1-\lambda)\tilde{y} & \text{in the progressive region.} \end{cases} \tag{1}$$

Figure 1: Employment incentives of progressive pensions



2.1 Employment incentives of different pension systems

We now investigate how different pension systems shape the incentives to work. These systems will only differ in the calculation formula $F(y_{20}, y_{21}, \ldots, y_{j_r-1})$ that relates an individual's earnings record to her pension payment.

[2]A value of $\lambda = 0$ resembles a proportional pension, whereas under $\lambda = 1$ the pension system would be fully flat beyond the bend point $b\bar{y}$.

Proportional system Our benchmark case is a purely proportional system in which

$$F(y_{20}, y_{21}, \ldots, y_{j_r-1}) = \frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20}.$$

This means that the individual pension is proportional to the individual's lifetime average earnings. The net return to working at age j under such a system is

$$w_{j,net}^{\mathrm{PR}} = (1 - \tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20}, \tag{2}$$

as each year of work at wage w_j increases the earnings record by $\frac{w_j}{j_r-20}$.

Lifetime-Earnings-Based Progressive Pensions In a lifetime-earnings-based progressive pension, the progressive formula $f(\cdot)$ is applied to *lifetime average earnings*. We consequently have

$$F(y_{20}, y_{21}, \ldots, y_{j_r-1}) = f\left(\frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20}\right)$$

and the net return to working at a given age j reads

$$w_{j,net}^{\mathrm{LE}} = (1 - \tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20} \times \boldsymbol{E_j}\left[\boldsymbol{f'}\left(\frac{\sum_{\boldsymbol{j=20}}^{\boldsymbol{j_r-1}} \boldsymbol{y_j}}{\boldsymbol{j_r - 20}}\right)\right]. \tag{3}$$

Compared to the proportional system, see equation (2), the incentives embedded in this pension system crucially depend on the individual's expected lifetime earnings and on the *marginal replacement rate* $f'(\cdot)$ of the pension calculation formula. If the individual expects to have lifetime average earnings that fall into the subsidy region $[0, b\bar{y}]$, then we get $f'(\cdot) = \frac{\lambda}{b} + (1 - \lambda) > 1$ and the incentives to be employed are larger than under a proportional system. In this case, the individual receives an *implicit employment subsidy*. If, however, the individual expects to end up in the progressive region of the system, the earnings of an additional working year will only increase pension benefits at rate $1 - \lambda < 1$. This creates a negative incentive to work. The dashed line in the right panel of Figure 1 visualizes the employment incentives of a lifetime-earnings-based progressive pension relative to a proportional pension as a function of the individual's *expected lifetime average earnings*. The employment incentives clearly jump from positive to negative at the bend point $b\bar{y}$. Consequently, the lifetime-earnings-based progressive pension incentivizes employment only for individuals with expected lifetime average earnings below the bend point $b\bar{y}$ and it disincentivizes employment for everyone else.

Annual-Earnings-Based Progressive Pensions In an annual-earnings-based progressive pension, the progressive pension formula $f(\cdot)$ is applied to *annual earnings* y_j. Consequently, we have

$$F(y_{20}, y_{21}, \ldots, y_{j_r-1}) = \frac{\sum_{j=20}^{j_r-1} f(y_j)}{j_r - 20}$$

and the net return to working becomes

$$w_{j,net}^{\text{AE}} = (1-\tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r - 20} \times \frac{\boldsymbol{f(w_j)}}{\boldsymbol{w_j}}. \tag{4}$$

Here, the net return to working increases for all individuals with an *average replacement rate* $\frac{f(w_j)}{w_j} > 1$. As shown in the left panel of Figure 1, the average replacement rate of a progressive pension exceeds that of a proportional pension for everyone who earns less than the economy's average earnings, i.e. $w_j < \bar{y}$. Hence, all those individuals receive an implicit employment subsidy compared to the proportional system. The solid line in the right panel of Figure 1 visualizes the employment subsidy as a function of the individual's *current annual earnings*. The implicit employment subsidy is the highest in the subsidy region $[0, b\bar{y}]$. It then fades out until a value of $\bar{y}$ and turns negative thereafter. Consequently, an annual-earnings-based progressive pension incentivizes employment for all individuals with below average annual earnings $\bar{y}$, and it (gradually) disincentivizes it for higher earners. Expected lifetime earnings play no role in determining these employment incentives.

2.2 Lifetime vs. Annual Earnings Redistribution

Real world pension systems, such as US Social Security, typically use a lifetime-earnings-based redistribution scheme. Yet, our simple model calculations suggest that choosing annual earnings as basis for old-age income redistribution can lead to much more favorable labor market outcomes, especially for below-average earners. This is true regardless of whether an individual has permanently low earnings or whether a low earnings episode is only temporary. In a lifetime-earnings-based system, the employment incentives are generally worse for a large part of the population. In particular, workers with a short-term adverse productivity shock experience a tax on their employment decision, as long as their expected lifetime earnings are larger than the bend point of the progressive pension formula.

In the context of our introductory discussion about female labor supply and women's old-age poverty risk, a proper design of the pension system can play a crucial role. A progressive pension system in which income redistribution is based on annual earnings could, for instance, attract women into the labor force during the years they are raising children. A continued work history offers additional income security at old ages, even beyond the regular insurance provided by redistributive pension systems. As such, an annual-earnings-based progressive pension has the potential to generate a superior income distribution at retirement than a lifetime-earnings-based one, in which women with young children are particularly discouraged from taking up work.

Lifetime-based redistribution should, however, allow for a better targeting towards workers permanently in need of funds. As such, it can separate redistribution and insurance concerns, which are inherently mingled when annual income is used as a redistribution base. This creates a trade-off between labor supply incentives and redistribution goals. In the remainder of this paper, we will study both the incentive and redistributive aspects of progressive pension systems in a quantitative simulation model. We consider men and women, single and couple households, and individuals with and without children. Our focus is on population groups who are most in need of resources in old age as well as those who are most prone to labor market incentives.

3 The Quantitative Simulation Model

Our quantitative analysis is motivated by the fact that empirical life-cycle labor supply patterns are quite distinct across demographic groups. Figure 2 shows data from the 2017 German Microcensus[3] on the distribution of labor hours for women and men over the life cycle. Women's labor supply is majorly shaped by family circumstances. Many women stop working full time when they have children in their late 20s and 30s. They rarely return to full-time work as their children grow older. About a quarter of all single women is not employed or has a minijob – a tax-advantaged, low-hour contract that often only pays the minimum wage. Another quarter works part time and the remainder 50 percent have a full-time contract. As a result, many single women receive low primary earnings during their working lives, which translate into low pension incomes and a higher old-age poverty risk. For married women, non-employment exhibits a wave-shape over the life cycle. As their children grow older, most of them return to work, but only on a part-time contract or in a minijob. Consequently, both young and old married women heavily dependent on their partner's income. Men show a much more stable pattern of labor supply. They generally work full time and non-employment is rather low until the age of 55. It continuously rises thereafter as men approach retirement.

Our full quantitative simulation model intents to speak to these empirical observations. It is populated by overlapping generations of households that can be distinguished according to a rich set of demographics. At the beginning of their (economically active) life, individuals are endowed with a gender and an education level. Based on these characteristics, they may get married to a partner or stay single. Throughout their life cycle, individuals are exposed to idiosyncratic labor productivity shocks. In addition, based on their partnership status and education,

[3] RDC of the Federal Statistical Office and Statistical Offices of the Federal States (2021)

Figure 2: Empirical labor supply

Data source: German Microcensus (2017).

they may give birth to children. Children cause both time and monetary costs to parents. Households are exposed to survival risk, especially during retirement. We assume that married partners die together.

Households decide about how much to work, consume and save. Decisions in a couple are made jointly, and partners can choose how to allocate labor hours across individuals. The labor supply decision features both an extensive and an intensive margin choice. Staying at home or working part time today may have intertemporal spill-overs in the sense that an individual's option to work full time in any future period may be restricted.

The government operates a pay-as-you-go pension system financed by payroll taxes. In addition, it collects revenue through the progressive taxation of labor earnings and a proportional consumption tax to cover general government expenditure and transfer payments to families with kids. We consider an open economy framework, so that the prices for capital and labor are fixed, but government parameters adjust in order to keep the fiscal tax and transfer systems balanced.

In the following, we provide a detailed description of model assumptions and equations. In addition, we immediately discuss parameter choices. Our base year is 2017, in which average earnings amounted to EUR 37,000, see DRV Bund (2020). Since we only consider long-run equilibria, we omit the time index t wherever possible. Owing to the richness of the model, we will remain very brief on the choices of parameters that can be considered as "standard" in the quantitative life-cycle model literature. Appendix C provides a detailed and thorough description of parameter choices as well as their empirical targets.

3.1 Demographics

Age, gender, education and marriage The economy is populated by overlapping generations of heterogeneous individuals, like in Auerbach and Kotlikoff (1987). At each point in time t, a new generation is born. We normalize cohort sizes to 1. Individuals start their economic life at age $j = 20$ and we allow for a maximum life span of $J = 99$ years. They enter the economy as either male or female $g \in \{m, f\}$ with either high-school or college education $e \in \{0, 1\}$. Based on these characteristics, they are potentially matched with a partner of the opposite gender to form a married couple. Marriages are stable over the life cycle and couples die jointly. If not matched to a partner, individuals form a single household. We denote the household type (single or couple) by $i \in \{s, c\}$. Gender, education and marital status constitute the permanent household characteristics.

We use data from the age cohorts 35-49 of the 2017 German Microcensus to estimate the following demographic parameters:

1. 50.78% of individuals in the sample are male, 33.06% of them have a college education, and 67.54% of them live in a couple household.[4]
2. The proportion of women is 49.22% and 27.76% of them have a college education.
3. We find a considerably degree of assortative mating, meaning that 85.69% of non-college educated men are married to a non-college educated woman and 54.81% of college-educated men are married to a college-educated woman.

We choose our model parameters to be consistent with these observations, see Appendix C.1.

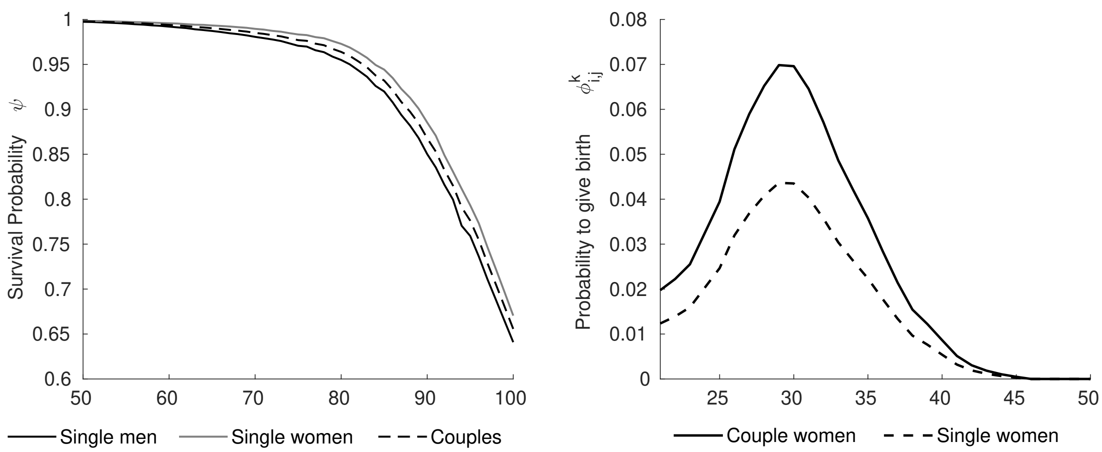
Retirement and survival Individuals can supply labor to the market until they reach the mandatory retirement age $j_r = 64$. This corresponds to the average retirement age of the German population in 2017, see DRV Bund (2019). Whether a household is still alive in the next period is uncertain and depends on survival probabilities that are specific to age, gender and marital status. We denote by $\psi^i_{j,g}$ the conditional probability of a household to survive from period $j-1$ to period j, with $\psi^i_{20,g} = 1$ and $\psi^i_{J+1,g} = 0$. We directly extract survival probabilities of singles from the 2017 annual life tables of the Human Mortality Database (2020) and use the average survival probability across genders for couples, see the left panel of Figure 3. Single men die at age 79.5 on average, single women at age 84.1 and couples at age 81.7. Cohort sizes shrink with age. We let m_j denote the relative size of the cohort aged j.

The arrival and presence of children Motherhood and child rearing are represented by the state $k \in \{0, 1, 2, 3\}$. Women start their life childless ($k = 0$). Between the ages 21 and 45 a woman may give birth to two children who are born

[4] This includes individuals who live in a couple household but are not formally married.

in the same period. Fertility is no active choice in our model, but children arrive with an age- and family-status specific child-birth probability $\phi^k_{i,j}$. According to the 2017 German Microcensus, 80.02% of married women and 47.53% of single women in the cohort 35-49 had at least one child present in their household. We use these numbers as proxies for the overall likelihood of giving birth to children. Furthermore, we use data from Eurostat (2023) on mothers' age at first birth to infer age-specific child-birth probabilities. Figure 3 (right panel) shows the resulting child-birth probabilities.

Figure 3: Survival probability and probability to give birth by age



Data sources: left: Human Mortality Database (2020), right: German Microcensus 2017, Eurostat (2023).

Fertility and child-rearing are modeled as a Markov process. Once a women gives birth, she transitions to state $k = 1$ which indicates the presence of young children (aged 0-5). After an average of 6 years in state $k = 1$, she transitions into $k = 2$, indicating the presence of older children (aged 6-17). After on average 12 more years, children leave the household and mothers are assigned the absorbing state $k = 3$.[5] At the retirement age j_r, mothers transition to state $k = 3$ with certainty. The Markov process is consequently governed by transition probabilities $\pi_k(k^+|k, j, i, g)$ that depend on age j and marital status i. In particular, the transition matrix for working-age women reads

$$\pi_k(k^+|k, j, i, f) = \begin{bmatrix} 1-\phi^k_{i,j} & \phi^k_{i,j} & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{11}{12} & \frac{1}{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{5}$$

[5] Kids never enter the economy as productive agents.

3.2 The Structure of the Labor Market

Labor hours choices Every individual has a time endowment equal to 1. Men can either choose to work full time (ℓ_{full}) or not at all.[6] Women, on the other hand, can choose from a menu of working hours $\{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$. ℓ_{part} corresponds to part-time work and ℓ_{mini} represents a minijob, a special form of marginal employment under which workers are allowed to earn 450 Euros tax free. According to the German Microcensus, full-time employees on average work for a fraction $\ell_{full} = 0.403$ of their total time endowment, whereas part-time employees work $\ell_{part} = 0.210$, see Appendix C.2.1. For minijobs, we set $\ell_{mini} = 0.1$ paying tribute to the fact that those jobs are typically low-hours marginal types of employment.

Labor market flexibility If a woman chooses not to work full time, this may restrict her future hours choices. We denote by h a woman's state of labor market flexibility at a given age. If $h = \ell_{full}$, then she can choose from the entire set of available hours. If $h = \ell_{part}$, then she is restricted in her choices and can only work part time, marginally or not at all. By modeling restricted working hours, we recognize that many women – and especially mothers – continue to work part time at older ages, even after their children have left the household, see for example Gallego Granados et al. (2019). We model the transition of h over ages as a first-order discrete Markov process with transition probabilities $\pi_h(h^+|h, g, \ell)$. Full-time working women remain in state $h = \ell_{full}$ with probability one. Women who do not work full time transition from $h = \ell_{full}$ into the state $h = \ell_{part}$ with a likelihood of 0.95. Once in this state, they come back to $h = \ell_{full}$ with an annual probability of 0.15. The average duration of a period of labor hours inflexibility is therefore 6.67 years. The parameter values are chosen such that the fraction of women working part time or less coincides with the empirical observations in Figure 2.

General labor productivity Households are ex ante homogeneous, but differ ex post in their labor productivity. All individuals of a given education level e share a common deterministic age-specific labor productivity profile $\theta_{j,e}$. Throughout working life, they are subject to idiosyncratic productivity shocks η, which follow a standard AR(1) process in logs

$$\eta^+ = \rho_e \eta + \varepsilon^+ \quad \text{with} \quad \varepsilon^+ \sim N(0, \sigma_{\varepsilon^2,e}), \tag{6}$$

where innovations ε^+ are iid across and within households. $\pi_\eta(\eta^+|\eta, e)$ denotes the probability distribution of next-period's productivity η^+, conditional on current labor productivity η and education e. We denote by $z(j, e, \eta) = \exp(\theta_{j,e} + \eta)$ the general productivity level of an individual at age j, education e and labor

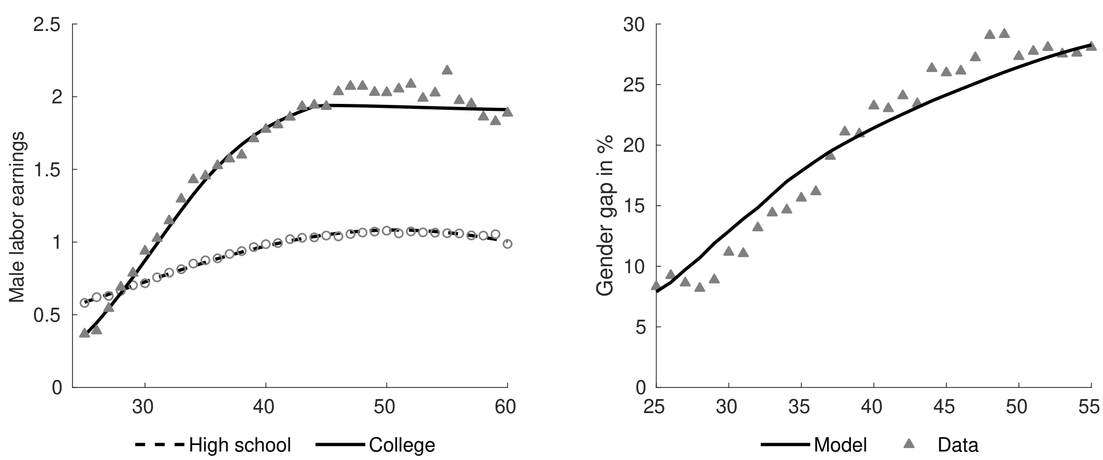
[6] In addition to computational ease, this choice is grounded in the empirically negligible share of men working part time, see Figure 2.

productivity shock η. A man's wage is then simply the product of the marginal product of labor w and general productivity, i.e.

$$w(j, e, \eta, m, 0) = w \times z(j, e, \eta).$$

We estimate the life-cycle labor productivity profiles $\theta_{j,e}$ as well as the idiosyncratic productivity risk process η from administrative data – the scientific use file of the Versichertenkontenstichprobe 2017 (FDZ-RV, 2017b) – on male earners from the German public pension insurance (Deutsche Rentenversicherung). On these data, we first run a regression with time and age fixed-effects to recover the average life-cycle labor earnings profile. We then use the residuals to parameterize the stochastic component of labor productivity. The left panel of Figure 4 shows the empirical and the model simulated life-cycle labor earnings profiles for men. Table 1 summarizes the parameters for labor productivity risk. The processes for labor productivity risk are highly persistent, with a somewhat smaller persistence for high-school workers than for college graduates. The overall unconditional process variance ranges at around 28 to 30 log-points. Appendix C.2.2 provides more details on the estimation procedure.

Figure 4: Empirical and model implied average life cycle earnings profiles



Data sources: left: FDZ-RV (2017b), right: Schrenker and Zucco (2020).

Gender wage gap Women face a lower wage level owing to a general gender wage gap w_{gap}. In addition, following evidence in the empirical literature, see for example Kleven et al. (2019), we assume that there is a motherhood wage penalty $w_{pnlty}(k)$ that depends on a woman's motherhood state k. Consequently, the wage a woman earns at any given state is

$$w(j, e, \eta, f, k) = w \times w_{gap} \times w_{pnlty}(k) \times z(j, e, \eta).$$

Table 1: Parameter values of labor productivity risk

	High school $e=0$	College $e=1$
Autocorrelation ρ_e	0.9300	0.9900
Innovation variance $\sigma^2_{\varepsilon,e}$	0.0372	0.0059
Unconditional variance $\frac{\hat{\sigma}^2_{\varepsilon,e}}{1-\hat{\rho}^2_e}$	0.2756	0.2983

Strictly speaking, this means that a woman is less productive than a man with the same age-education-productivity-shock combination. In reality, of course, there are several explanation for varying wage levels between men and women as well as mothers and non-mothers. This includes taste-based discrimination, less opportunities for human capital accumulation during child-rearing years, career ladder aspects or selection into different jobs based on individual fertility preferences as in Adda et al. (2017). Including all these channels, however, would go well beyond the scope of this paper.

The gray triangles in the right panel of Figure 4 show the gross wage gap between employed men and women over the life cycle as estimated by Schrenker and Zucco (2020). To account for these age-specific wage differences, we calibrate $w_{gap} = 0.87$ as well as a motherhood wage penalty that depends on the presence and age of children as

$$w_{pnlty}(\cdot) = \begin{bmatrix} 1.00 & 0.92 & 0.75 & 0.62 \end{bmatrix}.$$

Note that the *motherhood penalty* estimated in empirical studies typically combines the effects of hours and wage changes into one statistic. In contrast, our estimates of the pure *motherhood wage penalty* solely focus on wage differentials between mothers and non-mothers, which rather are a consequence of missed opportunities for accumulating specific human capital or climbing the career ladder. As such, the motherhood wage penalty rises with the age of children and is the largest when children have left the house. Using these estimates, our model provides a good fit for the gender gap ($w_{gap} \times w_{pnlty}(\cdot)$) over the life cycle, see the right panel of Figure 4. In Section 4.2 we also provide model simulations for the entire motherhood penalty that combines wage and hours differentials.

Labor earnings Labor earnings are finally calculated from a person's wage and his or her working hours

$$y_{j,g} = w(j, e, \eta, g, k) \times \ell_{j,g}.$$

In addition to regular working contracts, German tax and social security law allows for a special type of marginal employment, so-called minijobs. These jobs typically feature a small number of working hours and often only pay the minimum wage. Furthermore, workers can not earn more than a certain amount $\bar{y}_{mini}$. In turn, such jobs are exempt from income taxation and subject to reduced social contributions. For workers in this marginal employment category, we calculate labor earnings as

$$y_{mini,j,g} = \min\left[w(j,e,\eta,g,k) \times \ell_{mini}\ ,\ \bar{y}_{mini}\right]$$

and set their regular earnings to zero. Note that this formulation means that high productivity individuals can not work at their full labor productivity in such jobs. According to German tax law, there is an earnings-threshold for minijobs of 5400 Euros annually. Since not every minijob worker earns the maximum amount, we assume minijob earnings of 400 Euros per month which corresponds to 4800 Euros annually or $\bar{y}_{mini} = 0.1297 \times \bar{y}$. $\bar{y}$ denotes average earnings of the employed.

3.3 Preferences and the Budget Constraint

Preferences Individuals have preferences over stochastic streams of consumption $c_{j,g} \geq 0$ and labor supply $\ell_{j,g} \geq 0$. Single households maximize the discounted expected utility

$$U_0^s = E_0\left[\sum_{j=20}^{J} \psi_{j+1,g}^s \beta^{j-19} u(c_{j,g}, \ell_{j,g}, \xi)\right],$$

and couple households maximize the sum of individual discounted utilities. Expectations are formed with respect to future labor productivity, the future labor market flexibility of women, labor force participation costs, as well as the presence of children. Households discount the future with the constant time discount factor β as well as their individual survival rate.

We assume a period utility function

$$u(c_{j,g}, \ell_{j,g}, \xi) = \frac{c_{j,g}^{1-\sigma}}{1-\sigma} - \nu_g \frac{\left(\zeta_{k,i,g} + \ell_{j,g}\right)^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell_{j,g}>0}. \tag{7}$$

Utility is additively separable in consumption $c_{j,g}$ and labor supply $\ell_{j,g}$. Utility from consumption features constant absolute risk aversion σ, utility from labor a constant but gender-specific Frisch elasticity χ_g. Participation in the labor market is costly. Specifically, when choosing labor hours greater than zero, a worker has to pay the participation utility costs ξ. We assume that ξ is drawn at the household-level. This means that it is common to married couples but iid across households and across time and it is independent of individual labor productivity. We let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2.

We assign a value of 2 to risk aversion σ, a choice quite typical for the heterogeneous agent macroeconomics literature though at the lower end of values that generate an extensive desire for redistribution.[7] The empirical literature has pointed to the fact that Frisch elasticities differ significantly between men and women, see for example Keane (2011). Consistent with this evidence, we chose values of $\chi_m = 0.4$ for men and $\chi_f = 0.75$ for women. After making these data-based choices, we are left with the set $(\beta, \nu_m, \nu_f, \mu_\xi, \sigma_\xi^2)$ of parameters that we need to calibrate. We choose the time discount factor $\beta = 0.9785$ so that all capital is entirely absorbed by private savings in the initial equilibrium. We then jointly calibrate $\nu_m = 70$, $\mu_\xi = 1.65$ and $\sigma_\xi^2 = 2.5$ to match the participation rates of men across demographic groups in Table C.1 in Appendix C.2. Finally we set $\nu_f = 22.0$ to achieve an overall good divide between minijob, part-time and full-time work for women.

Families and children Families enjoy economies of scale in consumption.[8] Aggregate household consumption expenditure can be calculated from

$$c = c_g \times \upsilon(j, k, i),$$

where $\upsilon(j, k, i)$ is a scale factor that depends on the age and the composition of the household. We apply the new OECD equivalence scale.[9] This means that in order to realize the same individual consumption level, larger families face a smaller per capita spending. Children need to be fed. Consequently, they exhibit consumption costs through the scale factor $\upsilon(j, k, i)$.[10]

Since children must also be raised, their presence comes with time costs $\zeta_{k,i,g}$ that depend on the children's age. Time costs are fully borne by single mothers, but they can be partly shared by married couples. We calibrate the time cost of young children $\zeta_{1,s,f} = 1.50$ and older children $\zeta_{2,s,f} = 0.25$ to match the labor supply patterns of both single women as well as young women in Table C.1 in Appendix C.2. In order to match the empirical labor supply profiles of married women, too, we assume that the overall time costs of raising children are the same in single and married couple families, but that fathers take a certain (small) share of these costs. This leads us to $\zeta_{1,c,f} = 1.20$ and $\zeta_{1,c,m} = 0.30$ for young children as well as $\zeta_{2,c,f} = 0.1875$ and $\zeta_{2,c,m} = 0.0625$ for older children.

[7] In this model, σ fulfils two roles as it defines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter typically range between values of 1 and 3, whereas risk aversion can be quite high and well beyond values of 10 when estimated from individual financial choices, see for example Vissing-Jørgensen and Attanasio (2003).

[8] We refer to a household with more than one member as a family. Families can take the form of single-mothers, couples and couples with children.

[9] Each member of the household is given an equivalence value: 1.0 to the first adult, 0.5 to the second and 0.3 to each child. We don't distinguish between young and old children.

[10] Notation: Variables without a subscript denote household aggregates, like aggregate consumption expenditure c. Variables with a subscript refer to individual level units, like individual consumption c_g. We apply the same notation to earnings y, pensions p, bequests b, etc.

Budget constraint Markets are incomplete. Like in Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994), households can only self-insure against fluctuations in individual states by saving in a risk-free asset a with return r. They cannot borrow, so that assets must satisfy $a \geq 0$. A household's resources are composed of current wealth a (including returns), income from working in regular jobs y or in marginal employment y_{mini}, pension payments p, government transfers $t(k,i)$, and intergenerational transfers b. They use these resources to finance consumption expenditure $(1+\tau_c)c$ including consumption taxes, savings into the next period a^+, contributions to social security $T_p(\cdot)$, and progressive income taxes $T(\cdot)$. Consequently, the household budget constraint reads

$$\begin{aligned}(1+\tau_c)c + a^+ + T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big) + T\Big(y_m, y_f, p, i\Big) \\ = (1+r)a + y + y_{mini} + p + t(k,i) + b. \quad (8)\end{aligned}$$

Single households receive only one labor income, such that either y_m or y_f is equal to zero. Note that pension contributions are collected on the individual level and income taxes on the household level, see below.

3.4 Dynamic Optimization Problems

Singles The current state of a household with a single adult person is described by a vector $\mathbf{x_s} = (j, g, e, \eta, h, \xi, k, a, d)$ that summarizes the household's age j, gender g, education e, her current labor productivity shock η, her labor market flexibility h, her employment costs ξ, the presence and age of kids k, her wealth position a as well as the current amount of lifetime earnings d insured under the pension system. The dynamic optimization problem of a single household reads

$$v(\mathbf{x_s}) = \max_{c,\, \ell \leq h,\, a^+ \geq 0,\, d^+} \; u(c, \ell, \xi) + \beta \psi^s_{j+1,g} E\Big[v(\mathbf{x_s^+}) \;\Big|\; \mathbf{x_s}\Big] \tag{9}$$

with $\mathbf{x_s^+} = (j+1, g, e, \eta^+, h^+, \xi^+, k^+, a^+, d^+)$. Households maximize (9) subject to the budget constraint (8), the accumulation equation for lifetime insured earnings (14) as well as the laws of motion for children k, utility costs ξ, the labor choice set h, and labor productivity η. The result of this dynamic program are policy functions c, ℓ, a^+, and d^+ that all depend on the household's current state $\mathbf{x_s}$.

Couples The current state of a household with two married adults is described by a vector $\mathbf{x_c} = (j, e_m, e_f, \eta_m, \eta_f, h, \xi, k, a, d_m, d_f)$. It summarizes the joint household states age j, the labor market flexibility of the female partner h, the employment costs ξ, the presence and age of kids k, and household wealth a. In addition, it contains the individual specific education levels e_m, e_f, labor productivity shocks η_m, η_f, as well as the balance on individual pension accounts d_m, d_f for husband and wife, respectively. The dynamic optimization problem of a couple reads

$$v(\mathbf{x_c}) = \max_{\substack{c_m, c_f, \ell_m, \ell_f \leq h, \\ a^+ \geq 0, d_m^+, d_f^+}} \Big[u(c_m, \ell_m, \xi) + u(c_f, \ell_f, \xi)\Big] + \beta \psi^c_{j+1} E\Big[v(\mathbf{x_c^+}) \;\Big|\; \mathbf{x_c}\Big] \tag{10}$$

with $\mathbf{x_c}^+ = (j+1, e_m, e_f, \eta_m^+, \eta_f^+, h^+, \xi^+, k^+, a^+, d_m^+, d_f^+)$. We provide an analytical derivation of the household's first-order conditions in Appendix B.1.

3.5 Technology

A continuum of identical firms produce a single good Y under perfect competition. They hire both capital K at price r and labor L at price w on competitive spot markets. Firms operate a constant returns to scale technology

$$Y = \Omega K^{\alpha} L^{1-\alpha}. \tag{11}$$

Ω denotes the aggregate level of productivity, whereas α is the elasticity of output with respect to capital. In the process of production, a fraction δ of the capital stock depreciates. Given the assumptions about competition and technology, we can safely assume the existence of a representative firm that takes prices as given and operates the aggregate technology in (11). In addition to employing factor inputs, the firm has to invest I_t into its capital stock. The law of motion for the capital stock reads

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{12}$$

We choose a depreciation rate of $\delta = 0.07$, which leads to a realistic investment to output ratio of 21%, see German Statistical Office (2020). We set the capital share in production at $\alpha = 0.30$ to obtain a capital-to-output ratio of three and normalize the technology level Ω such that the wage rate per efficiency unit of labor w is equal to 1. Finally, we assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

3.6 The Pension System

We model a purely proportional pension system in the initial equilibrium. Progressive pension systems are discussed in Section 4.2. Unlike the tax system, the pension system operates on an individual basis. Each individual in a household is liable for her own contribution and accumulates her own pension entitlements. The pension system collects payroll taxes at rate τ_p on earnings from regular employment y_g as well as on a share ϱ of minijob earnings $y_{mini,g}$. Earnings are subject to payroll taxation up to a contribution ceiling equal to $2\bar{y}$. Consequently, *contributory earnings* $y_{con,g}$ and the payroll tax load T_p read

$$y_{con,g} = \min\left[y_g + \varrho \times y_{mini,g},\ 2\bar{y}\right] \quad \text{and} \quad T_p(y_g, y_{mini,g}) = \tau_p \times y_{con,g}.$$

In reward for their contributions, individuals are credited with a certain amount of *insured earnings* $y_{ins,g}$. Insured earnings consist of contributory earnings plus a pension top-up $p_{child}(1) = \bar{y}$ for mothers of young children ($k = 1$).[11] The top-up is meant to compensate mothers for their reduced earnings while raising young children. Earnings plus top-up can't exceed the contribution limit. Insured earnings in a given year consequently are

$$y_{ins,g} = \min\Big[y_{con,g} + p_{child}(k),\ 2\bar{y}\Big]. \tag{13}$$

Individuals accumulate their insured earnings to *lifetime insured earnings* d_g according to

$$d_g^+ = d_g + y_{ins,g}. \tag{14}$$

Note that during retirement d_g is constant as insured earnings $y_{ins,g}$ are equal to zero. *Pension benefits* p_g at retirement are proportional to the lifetime average of insured earnings

$$p_g = \kappa \times \frac{d_g}{j_r - 20}, \tag{15}$$

where κ denotes the replacement rate of the pension system.

The pension system runs on a pay-as-you-go basis. In equilibrium, total annual pension contributions equal the total amount of annual pension payments. We fix the pension contribution rate at its statutory rate of $\tau_p = 0.187$ in 2017. The accrual rate for minijob earnings is $\varrho = 0.80$. These choices result in a value of $\kappa = 0.41$.

3.7 The Tax System and Government Expenditure

The government collects proportional taxes on consumption expenditure at rate τ_c and operates a progressive tax on labor earnings y_g and pension payments p_g. Earnings from minijobs y_{mini} are tax free. Individuals can deduct their pension contributions for the purpose of taxation, i.e. taxable earnings are

$$y_{tax,g} = y_g - \tau_p y_{con,g}.$$

The income tax function reads

$$T(y_m, y_f, p, i) = \begin{cases} \mathcal{T}\Big(y_{tax,g} + p_g\Big) & \text{if } i = s \\ 2\mathcal{T}\Big(\frac{y_{tax,m}+y_{tax,f}+p_m+p_f}{2}\Big) & \text{if } i = c, \end{cases} \tag{16}$$

where $\mathcal{T}(\cdot)$ denotes the tax schedule. Couples enjoy tax benefits through the income splitting method. Hence, taxes are based on average household earnings.

We employ the 2017 statutory German progressive income tax code as depicted in Figure C.1 in Appendix C.6. In addition, we set the proportional consumption

[11] For all other individual, we have $p_{child}(k \neq 1) = 0$.

tax rate at $\tau_c = 0.16$ to balance the fiscal budget. Although consumption goods are regularly taxed at a rate of 19% in Germany, many goods (such as food, books and newspapers) are taxed at a lower rate. In our simulations, we assume that the consumption tax rates balances the fiscal budget on an annual basis.

Tax revenue is used to finance (wasteful) government consumption G and child-related transfers $t(k, i)$ to families with children. We set G to 19% of GDP in the initial equilibrium economy, see German Statistical Office (2020), and assume that it is fixed per capita. In 2017, parents received a child benefit of EUR 192 per child and month. Moreover, we let the government pay additional tax financed child support payments to single mothers, which mimic both alimony payments and subsistence transfers in the real world. We set these monthly child support payments to EUR 576 per child. Appendix C.6 provides additional details.

3.8 Capital Markets, Trade and Equilibrium

We model a small open economy that freely trades capital and goods on competitive international markets. All private savings that are not employed by the domestic production sector are invested abroad at the international interest rate $\bar{r}$, see Appendix B.3 for further details. The government collects all accidental bequests that households might leave if they die before the terminal age J. Bequests are redistributed in a lump-sum way among the surviving working-age population through the intergenerational transfer b. Given an international interest rate and the exogenous fiscal policy parameters, a *recursive competitive equilibrium* of this model is a set of household policy functions, a measure of households, optimal production inputs, factor prices, accidental bequests, a net foreign asset position and a trade balance that are consistent with individual optimization and market clearance. A formal definition of the equilibrium is available in Appendix B.3.

4 Simulation Results

This section presents simulation results from our quantitative model. We first investigate the initial equilibrium economy. The model is successful in replicating real-life data both on the macro and the micro level. Next, we turn to counterfactual policy simulations, in which we introduce redistributive components into the pension system.

4.1 The Initial Equilibrium Economy

The macroeconomy Table 2 summarizes central macroeconomic aggregates of the initial equilibrium and compares them to the data for 2017. The capital stock amounts to three times GDP both in the model and in the data. The discount factor β is calibrated such that private assets cover total capital demand of firms.

This is a solid approximation of reality, where private assets of the bottom 99 percent wealth holders are only slightly larger than the capital stock.[12] On the goods market, government consumption and investment are in line with the data. In the absence of a positive trade balance, private consumption in the model amounts to private consumption plus net exports in the data.[13] Consumption and labor tax revenue are somewhat higher than in the data, as revenues from corporate and property taxation are absent in our model. We perfectly match the pension contribution rate by construction and we decently fit the gross replacement rate of the average earner. Finally, our model is able to replicate the employment rates of women and men.

Table 2: Macroeconomic aggregates

Variable	Value	Data 2017
Capital stock	300.00	305.24
Private assets	300.16	332.70
Government consumption	19.00	19.84
Investment	21.00	20.96
Private consumption	60.00	52.11
Trade balance	0.00	7.09
Labor tax revenue	10.09	8.35
Consumption tax revenue	9.70	8.74
Pension contribution rate (in %)	18.70	18.70
Replacement rate of the avg. earner (gross, in %)	41.33	44.80
Employment rate women (ages 25-63)	75.71	73.68
Employment rate men (ages 25-63)	84.02	85.23
Total employment rate (ages 25-63)	79.92	79.35

Variables in percent of GDP if not indicated otherwise.

Data sources: PA: Alvaredo et al. (2022), CS: German Statistical Office (2020), PC, GC, I, TB, LTR, CTR: German Statistical Office (2020), pension data: DRV Bund (2020), labor data: German Microcensus (2017).

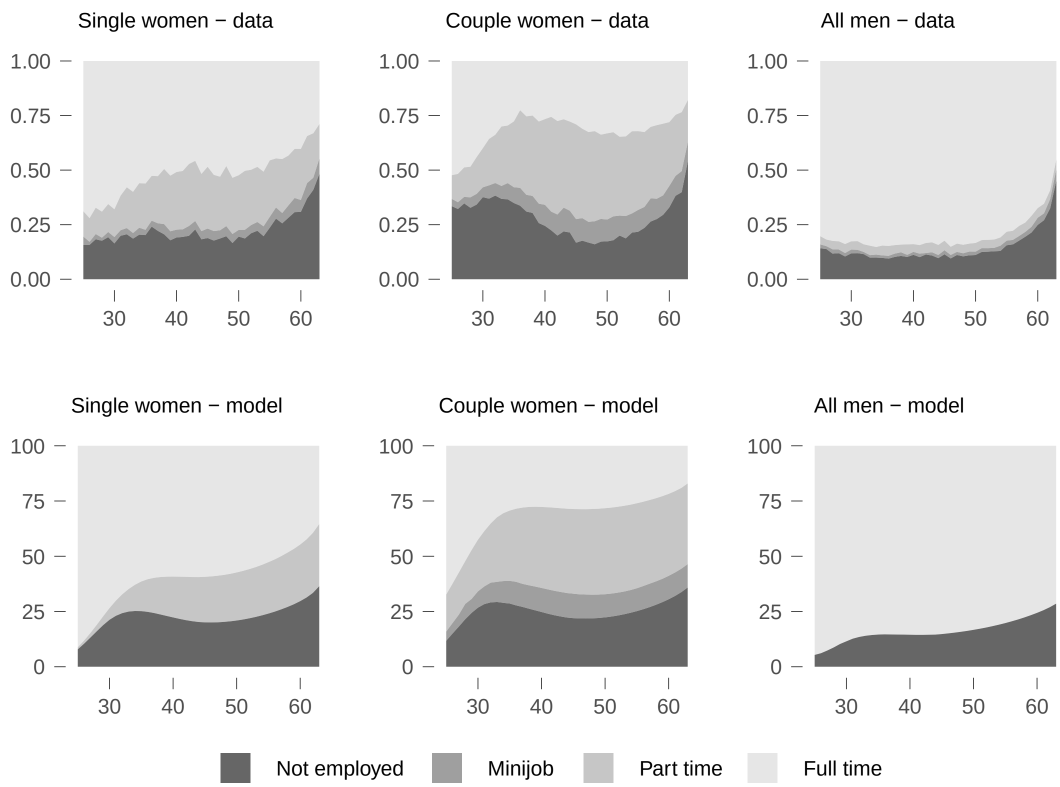
Labor supply profiles Figure 5 compares the empirical life-cycle labor supply profiles (first row) to the model implied counterparts (second row). The model is able to match the distinct labor supply patterns of single and married women. In particular, it replicates the wave-shaped pattern of non-employment that results from the arrival of children. A sizable fraction of women works part time or in a

[12] We exclude the top 1% wealth holders from the private asset data, as they are also not represented in our model.

[13] Note that Germany has both a positive trade balance and a positive net foreign asset position. In a long-run equilibrium, this is impossible to achieve without a permanently positive balance of payments. Hence, we decided to strike a balance by having both the net foreign asset position and the trade balance equal to zero.

minijob even at older ages when children have already left the household. As in the data, the pattern is more pronounced for married than for single women. For men, non-employment strictly rises with age. Finally, the simulation model somewhat over-predicts the share of full-time workers at young ages for all demographic groups. This may result from the fact that liquidity constraints are still prevalent for young households and that we are missing a set of intergenerational transfers. In terms of our policy exercise, which aims at stimulating employment through the pension system, this means that we may underestimate the potential for labor supply responses somewhat for younger households.

Figure 5: Empirical and simulated labor supply



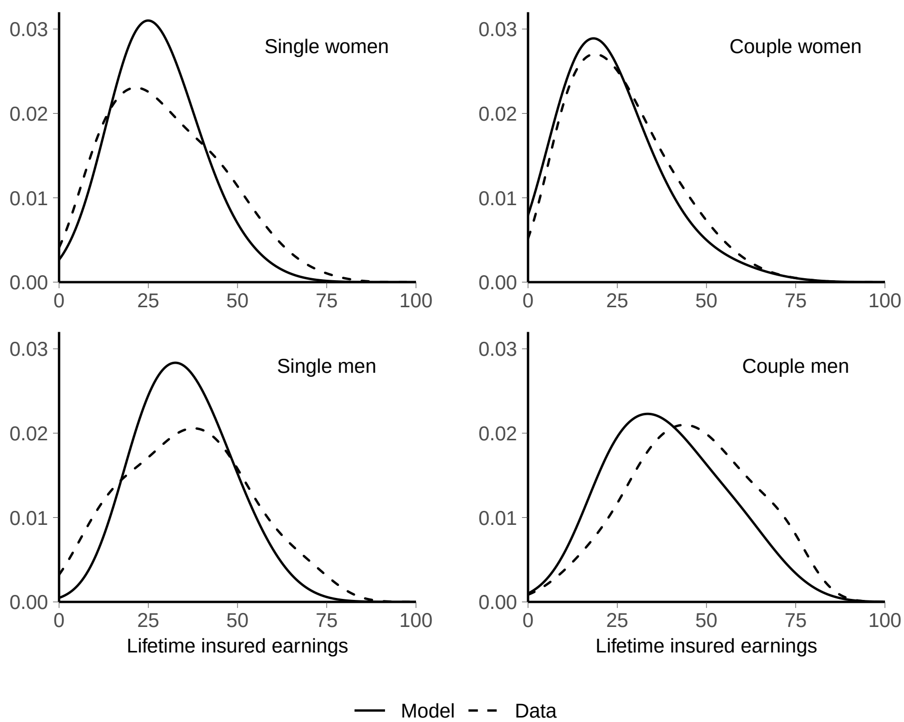
Data source: German Microcensus (2017).

The distribution of lifetime insured earnings In Figure 6, we contrast the empirical and the model simulated distributions of lifetime insured earnings d_g at retirement entry. Note that they were not targeted in the calibration process. To calculate the empirical distributions, we use administrative data from the German public pension insurance (FDZ-RV, 2017a).[14] We model women's labor supply

[14]We restrict our sample to individuals with a history of at least 180 pension relevant months at retirement entry. This includes relevant periods of non-work such as unemployment, care leaves, etc. In doing so, we account for the fact that a number of individuals chose to become

decisions very rigorously and thus generate a fairly good fit for their lifetime insured earnings. In particular, our model predicts a reasonable share of women at the lowest end of the earnings distribution. While a married woman with low or no pension wealth can rely on her husband, a corresponding single woman is at poverty risk in old age. The model implied distribution for men deviates somewhat from the data. The empirical distribution has a fatter left tail for singles and a fatter right tail for married men. A possible explanation for this may lie in the selection process into marriage. Although we account for assortative mating by education, the model does not consider other possible selection criteria. Pollmann-Schult (2011) discusses various reasons for why married men typically earn more than comparable single counterparts.

Figure 6: Distribution of lifetime insured earnings d_g



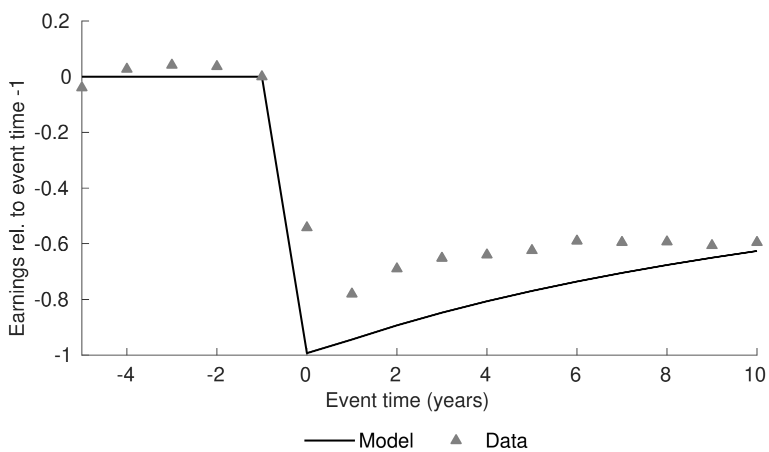
Data sources: FDZ-RV (2017a). Kernel density estimate with a Gaussian kernel and a bandwidth of 6.

The motherhood penalty Figure 7 shows the motherhood penalty in our model and compares it to data from Kleven et al. (2019). To make a fair comparison, we perform an event study analysis in the same way as Kleven et al. (2019). In particular, we measure the evolution of women's labor earnings starting from

self-employed or civil servants during their working lives and therefore may opt out of the public pension system. We estimate the distributions using a kernel density estimate with bandwidth equal to 6. We apply the same type of kernel density estimate to your model simulated data in order to make the empirical and model results comparable.

the date they gave birth to their first child ($t = 0$), relative to their earnings in the previous period ($t = -1$). The results shown in the figure are an average over the entire model population of women who gave birth to children, meaning that they mix single and married women at different points in their life cycle. Like in the data, child birth is accompanied by a large drop in earnings. The initial drop in earnings is, most certainly, driven by changes in female labor hours. Over time, earnings somewhat recover, but even after ten years a sizable 60 percent gap prevails. The long-run motherhood penalty then results from a mixture of lower working hours and the motherhood wage penalty, see Section 3.2. Overall, our model provides a decent fit for the empirical motherhood penalty. Naturally, the empirical estimates are somewhat fuzzy around the event date, as they are based on annual earnings and women may give birth to children throughout the year. Yet, our model is especially successful in replicating the medium- and long-run effects of motherhood on earnings.

Figure 7: Empirical vs. simulated motherhood penalty



Data sources: Kleven et al. (2019).

4.2 Counterfactual: Annual vs. Lifetime Redistribution

In our counterfactual analysis, we replace the status-quo proportional German pension system as outlined in Section 3.6 with a progressive one. To this end, we draw on the progressive pension formula discussed in the left panel of Figure 1 as well as in equation (1). We compare the effects of both an annual-earnings-based and a lifetime-earnings-based progressive pension, see the discussion in Section 2. We use a medium range progressivity parameter of $\lambda = 0.5$.[15] In addition, we

[15]Simulation results for alternative progressivity parameters are available upon request. While different choices of λ result in different quantitative effects, their qualitative implications are identical.

choose a bend point of $b = 0.3$. As we argue in another study, see Kindermann and Pueschel (2023), there is a trade off in choosing this bend point: From the perspective of economic theory, a small bend point is to be preferred, as it comes with the highest extensive margin labor supply incentives and therefore raises economic welfare. Yet in practice, having a bend point that is too small may lead workers to engage in very-low-hours or even fictitious working contracts just to benefit from the pension subsidy, which can again be detrimental to welfare. A value of $b = 0.3$ is a good compromise between these two objectives.

To ensure comparability between different simulations, we use the same set of structural parameters and fix per-capita government consumption at the initial equilibrium value. We assume further that the contribution rate of the pension system remains at the initial equilibrium level. In doing so, we ensure that the size of the pension system relative to total labor hours is constant for all reforms. The replacement rate κ is then used to balance the pension budget.[16] We compare results from long-run equilibrium outcomes only and neglect the transition path.[17]

Annual earnings redistribution In a system with *annual-earnings-based redistribution*, we apply the progressive pension formula $f(\cdot)$ to the calculation of annual insured earnings $y_{ins,g}$. Hence, the calculation formula for insured earnings (13) has to be adjusted. The pension benefit formula (15) remains unchanged relative to the proportional pension system. Specifically, we set

$$y_{ins,g} = f\Big(\min\big[y_{con,g} + p_{child}(k),\ 2\bar{y}\big]\Big) \quad \text{and} \quad p_g = \kappa \times \frac{d_g}{j_r - 20}.$$

Lifetime earnings redistribution In a system with *lifetime-earnings-based redistribution*, the calculation formula for the insured earnings (13) is the same as in the proportional system. The pension benefit formula (15) needs to be adjusted. We let

$$y_{ins,g} = \min\big[y_{con,g} + p_{child}(k),\ 2\bar{y}\big] \quad \text{and} \quad p = \kappa \times f\left(\frac{d_g}{j_r - 20}\right).$$

In the remainder of this paper we analyze whether and how the choice of annual versus lifetime earnings redistribution in a progressive pension system matters. Following the discussion in Section 2, we expect a system with annual earnings redistribution to incentivize labor force participation for a broad part of

[16] Note that, alternatively, we could fix total expenditure of the pension system at the initial equilibrium level. This is, however, counterfactual to the nature of a pay-as-you-go system. With fixed total expenditure, an increase in labor force participation or labor hours would lead to a decline in per capita pension payments and therefore lead to a cut in pension benefits which would counteract the positive effects of the proposed pension reforms.

[17] This choice is based on the complexity and demand for physical space of the model. In Kindermann and Pueschel (2023) we demonstrate in a simpler version of this model that taking into account the short-run effects of progressive pension reforms typically enforces our argument.

the working-age population. This includes workers with transitory low labor productivity shocks, mothers who are occupied by other tasks like caring for children, and those who approach retirement and have already built enough private savings to retire early. In fact, every worker with potential earnings below average earnings will enjoy an implicit employment subsidy, see the right panel of Figure 1. The participation incentives in a lifetime-earnings-based system, on the other hand, do not depend on instantaneous worker characteristics, but on (expected) lifetime average earnings only. In particular, workers will only enjoy an employment subsidy if they expect their lifetime average earnings to be below the bend point $b\bar{y}$ of the progressive pension formula. Consequently, a lifetime-earnings-based system can create the opposite type of incentives leading workers who are on the margin of non-participation to drop out of the labor force temporarily.

In addition to setting (positive or negative) participation incentives, both progressive systems will negatively distort the intensive margin labor supply decision. For workers with earning greater than the bend point $b\bar{y}$, the progressive pensions systems flatten the link between pension contributions and pension payments. This leads to a higher implicit tax rate on intensive margin labor supply. Finally, both progressive pensions can be expected to create a more equal distribution of pension payments. In this regard, a lifetime-earnings-based system may come at a more targeted level of redistribution between individuals permanently in need of resources and those who generally have enough to make a living on their own. To elaborate on all of these effects, we need a simulation model like the one presented above that has enough demographic detail to accurately quantify the labor supply responses and resource needs of a greatly heterogeneous population.

4.3 Extensive Margin Labor Supply Effects

Our simulation results show that the introduction of progressive pensions comes with distinct effects on aggregate employment. Aggregate employment increases under a system with annual-earnings-based redistribution, while it drops under lifetime-earnings-based redistribution. The effects are particularly pronounced for women. This is consistent with the discussion in the previous section as well as with our theoretical model in Section 2. In the following, we discuss how the employment effects differ by gender, age and productivity.

Employment effects by demographic groups Table 3 shows changes in employment separately for women and men. Women's employment decision seems to be more responsive to the incentives set by the two different progressive pension systems (see column "All"). The employment rate of women increases more than that of men in the annual-earnings-based system and it decreases more in the lifetime-earnings-based system. This stems from the fact that women have a generally lower productivity level and tend to be more often at the margin of being employed. In addition, the employment effects of both systems are more pronounced for single individuals than for married couples. Single individuals are directly exposed to the benefits and costs of a changing degree of redistribution in the pension system. As such, they immediately react to positive and negative work incentives. In couple households, decisions are made on a two-earner basis and, even if there are incentives to work an additional year, there may be good reasons not do so, for example because of a high degree of specialization. What is more, increased redistribution towards a secondary earner directly comes at the cost of declining pension payments for the primary earner in a married couple. This constitutes a negative income effect for the entire family. Put differently, what comes as redistribution between different households for singles may only be redistribution within the family for married couples.

Table 3: Changes in employment by population groups

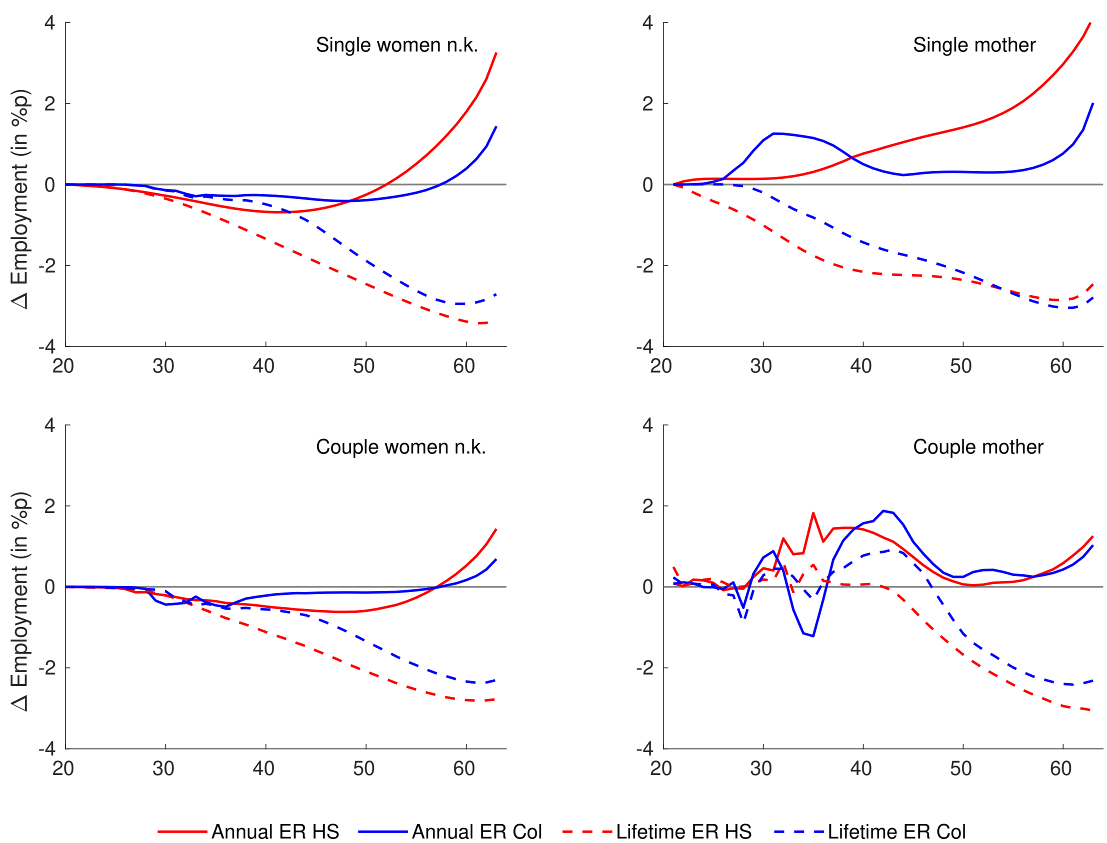
	All	**Singles**	**Couples**	**Ages: 20-44**	**Ages: 44-63**
			Women		
Annual ER	0.24	0.34	0.20	0.12	0.41
Lifetime ER	−1.12	−1.52	−0.95	−0.33	−2.21
			Men		
Annual ER	0.03	0.13	−0.02	0.02	0.05
Lifetime ER	−0.99	−1.13	−0.92	−0.31	−1.92

Percentage point difference from initial equilibrium with proportional pension system.
Employed population ages 20-63.

Table 3 also shows that employment effects are much more pronounced for older than for younger workers. This is because young workers have not accumulated a lot of assets yet. As such, extensive margin labor supply elasticities are typically smaller for younger than for older households, who have already accumulated a lot of savings and may decide to retire early as their productivity falls. An annual-earnings-based redistributive pension system counteracts this early retirement channel, leading to higher employment among elder workers. A system with lifetime earnings redistribution, on the other hand, enforces early retirement by setting negative employment incentives for all workers with medium to high lifetime average earnings.

Employment effects by age, family status and education Figure 8 provides a deeper insight into the extensive margin effect over the life cycle for women with different education and family status. The solid lines indicate changes resulting from the introduction of an annual-earnings-based pension system, while the dashed lines refer to a lifetime-earnings-based pension system. Red lines are for high school workers while blue lines refer to college graduates. Under annual-earnings-based redistribution, women without children (left column) reduce their employment marginally between the ages of 20 and 55. This is owning to the fact that they typically were full-time workers in the initial equilibrium and that the introduction of progressive pensions has diminished the return to working full time. At older ages, individuals often retire early when enough private savings are available. At this stage, the extra employment incentives set by an annual-earnings-based progressive pension kick in. As a result, the employment changes are significantly positive and can amount to almost four percentage points for women close to retirement. Mothers (right column) experience positive employment effects throughout their entire life cycle, since they often did not work in the initial equilibrium owing to their child-related duties. Finally, the employment effects towards the end of the life cycle tend to be stronger for women with a high school degree than for college graduates. This is explained by the different life-cycle productivity profiles of the two education groups, see Figure 4.

Figure 8: Changes in employment rates women



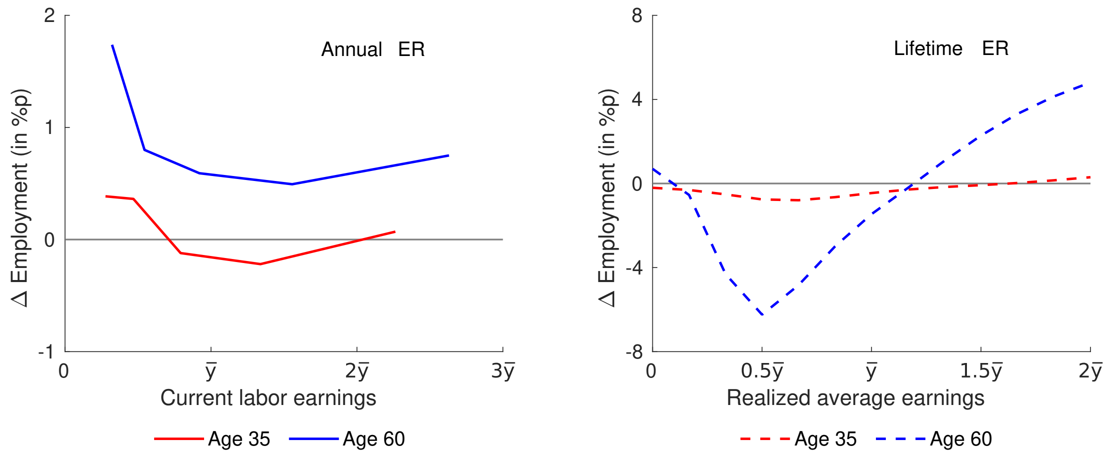
Notes: Percentage point difference from initial equilibrium with proportional pension system.

The picture looks quite different for a progressive pension in which redistribution is based on lifetime earnings. Here, the employment incentives are almost unanimously negative and the drop in employment becomes sharper as individuals approach retirement. Incentives in such a system are purely based on lifetime average earnings and not on instantaneous earnings. As individuals approach retirement and they realize that their lifetime average earnings d_g are high enough to bring them over the bend point $b\bar{y}$, their return to working diminishes and they may decide to retire early. The only exception from this are married mothers with a college degree in their late 30s and 40s. They are likely married to a college graduate husband. The family hence experiences a large negative income effect from the progressive pension reform. This may lead the wife to increase employment for some time to cover up for these losses. Simulation results for men are qualitatively similar and provided in Figure E.1 of Appendix E.

Employment incentives by (lifetime) earnings In Figure 9, we show how employment incentives in an annual-earnings-based (left panel) and a lifetime-earnings-based (right panel) progressive pension system depend on current earnings and lifetime earnings, respectively. In this case, we only look at single men, since they can only work full time if employed and they are not affected by a second earner. This makes the patterns we find particularly transparent. In Figure 1 we already discussed that an annual-earnings-based system sets the strongest employment incentives for individuals with (potential) current earnings below the bend point $b\bar{y}$. Yet, it also provides positive incentives for all workers up to average labor earnings $\bar{y}$. This can be readily seen from the left panel of Figure 9, which shows the change in employment for workers with different current earnings (expressed as multiples of average earnings $\bar{y}$). Employment changes are positive and the strongest for workers with very low earnings. As earnings increase, the positive employment effect fades out. Employment changes bottom out around the average earnings level and then rise slightly. The latter is due to the fact that individuals with above-average earnings lose a fraction of their old-age income and want to compensate for that by working more. This effect is particular pronounced for the elderly. In fact, our simulation results indicate that 60-year-olds increase their employment across the entire earnings distribution. Still, the effects are qualitatively in line with the incentive scheme of the pension system discussed in Figure 1.

The return to work in the lifetime-earnings-based system is solely determined by (expected) lifetime insured earnings d. An individual's realized earnings history up to the current age j is therefore a key determinant of employment decisions. This is shown in the right panel of Figure 9, which plots the change in employment as a function of the average lifetime earnings an individual has attained until age 35 and 60, respectively. We find that the 35-year-old generation reacts relatively little to the pension reform regardless of their past earnings history. This is not surprising as their realized earnings still provide very little information about their final lifetime insured earnings d. Yet, the response of 60-year-olds is quite in line with the incentive scheme described in Section 2. Those with very low lifetime earnings slightly increase their employment, and the employment incentives

Figure 9: Employment incentives by pension reforms for single men



Notes: Percentage point difference from initial equilibrium with proportional pension system.

become negative thereafter. Only individuals with very high lifetime earnings increase their employment to make up for a decline in their pension payments.

4.4 Intensive Margin Labor Supply Effects

Table 4 shows the intensive margin labor supply responses of women to the introduction of progressive pensions. Recall that women can choose between working full time, part time or in a minijob. The numbers presented are percentage point changes in the share of women that choose each of these three types of work contracts. Consequently, the three numbers sum up to the extensive margin employment effects presented in Table 3 for each demographic group.

Table 4: Intensive labor supply responses by population groups (women)

	minijob	**PT**	**FT**	**minijob**	**PT**	**FT**
	Singles: ages 20-63			Couples: ages 20-63		
Annual ER	0.00	2.12	−1.78	−1.77	3.66	−1.69
Lifetime ER	0.00	1.17	−2.69	−0.60	1.12	−1.47
	Young: ages 20-44			Older: ages 44-63		
Annual ER	−0.68	1.61	−0.82	−1.98	5.34	−2.95
Lifetime ER	−0.17	0.45	−0.61	−0.75	2.07	−3.52

Percentage point difference from initial equilibrium with proportional pension system.
Women ages 20-63.

The intensive margin labor supply patterns are qualitatively similar for all demographic groups and for both types of progressive pension systems. They are a direct consequence of the phase-in and phase-out structure of the progressive pension schedule presented in Figure 1. Pensions rise disproportionately for workers in the subsidy region, and working longer hours becomes more favorable for them. In the progressive region, however, the link between earnings and pension payments flattens, which increases the implicit tax on labor earnings for higher income earners. Hence, they work fewer hours. This incentive scheme is reflected in the results presented in Table 4. For all demographic groups, the number of workers in minijobs and the number of full-time workers declines, while the number of part-time workers increases. As minijobs only count towards an individual's insured earnings at a reduced rate, the decline in minijob work automatically leads to an expansion in pension coverage. The two-sided shift into part-time work is much more pronounced under a system with annual-earnings-based redistribution. Here, current earnings play a much stronger role in determining an individual's position in the pension schedule. In a system with lifetime-earnings-based redistribution, it is the individual's expected lifetime average earnings that determines intensive margin labor supply incentives. Hence, the intensive margin effects are less salient.

4.5 Impact on the Macroeconomy

Table 5 shows the macroeconomic consequences of the two pension reforms. Weekly working hours of employed women decline by about 0.30 hours per week under both reforms. With an annual earnings-based system, the adverse effect on aggregate labor supply is partly compensated by an increase in the employment rate of 0.13 percentage points. Hence, the decline in total labor input – and therefore aggregate capital and GDP – is limited to 0.36 percent. Under a lifetime-earnings-based pension system, a drop in the total employment rate by 1.06 percentage points reinforces the decline in aggregate labor supply. As a result, total labor input and therefore GDP fall by 1.59 percent. The decline in long-run GDP and the capital stock is accompanied by a reduction in aggregate consumption and investment, as it is typically found for fiscal reforms that come at increased redistribution and insurance. Progressive pensions reduce the need for precautionary behavior due to increased insurance. This lowers total private savings.

The fiscal consequences are much more moderate under annual-earnings-based than under lifetime-earnings-based redistribution. The decline in aggregate labor input comes at a short-fall of labor tax revenue which needs to be compensated by an increase in consumption taxes. Therefore, the consumption tax rate has to rise by 0.34 and 0.91 percentage points in the annual- and lifetime-earnings-based system, respectively. Aggregate pension payments relative to GDP fall in both reforms. Recall that we fixed the contribution rate to the pension system over time. As aggregate labor supply drops, total pension contributions fall and total pension payments have to adjust in the same direction. The replacement rate of

Table 5: Reform: macroeconomic aggregates

Variable	Annual ER	Lifetime ER
Avg. weekly hours of the employed women (ages 20-63)	−0.29	−0.31
Employment rate women (ages 20-63, in%p)	0.24	−1.12
Employment rate men (ages 20-63, in%p)	0.03	−0.99
Total employment rate (ages 20-63, in%p)	0.13	−1.06
GDP/capital stock/labor input	−0.36	−1.59
Private consumption	−0.61	−2.02
Investment	−0.36	−1.59
Private savings	−1.22	−1.11
Labor tax revenue	−1.25	−3.01
Consumption tax revenue	1.45	3.49
Consumption tax rate (in %p.)	0.34	0.91
Total pension payments	−0.41	−1.68
Replacement rate of the avg. earner (in %p)	−2.25	−6.48

Changes in percent over initial equilibrium with proportional pension system if not indicated otherwise.

the average earner[18] declines by 2.25 and 6.48 percentage points, respectively. The large difference cannot be explained solely by declining contributions. Instead, it shows that under a lifetime-earnings-based system the group of subsidy recipients is larger than under an annual-earnings-based system. In particular, it contains individuals with only few pension contributions, like women who never return to work after having children. Under annual earnings redistribution, being employed is a prerequisite for receiving a subsidy. As such, the ratio between contributors and beneficiaries is more favorable, and more resources are left for the average earner.

4.6 Distributional Effects

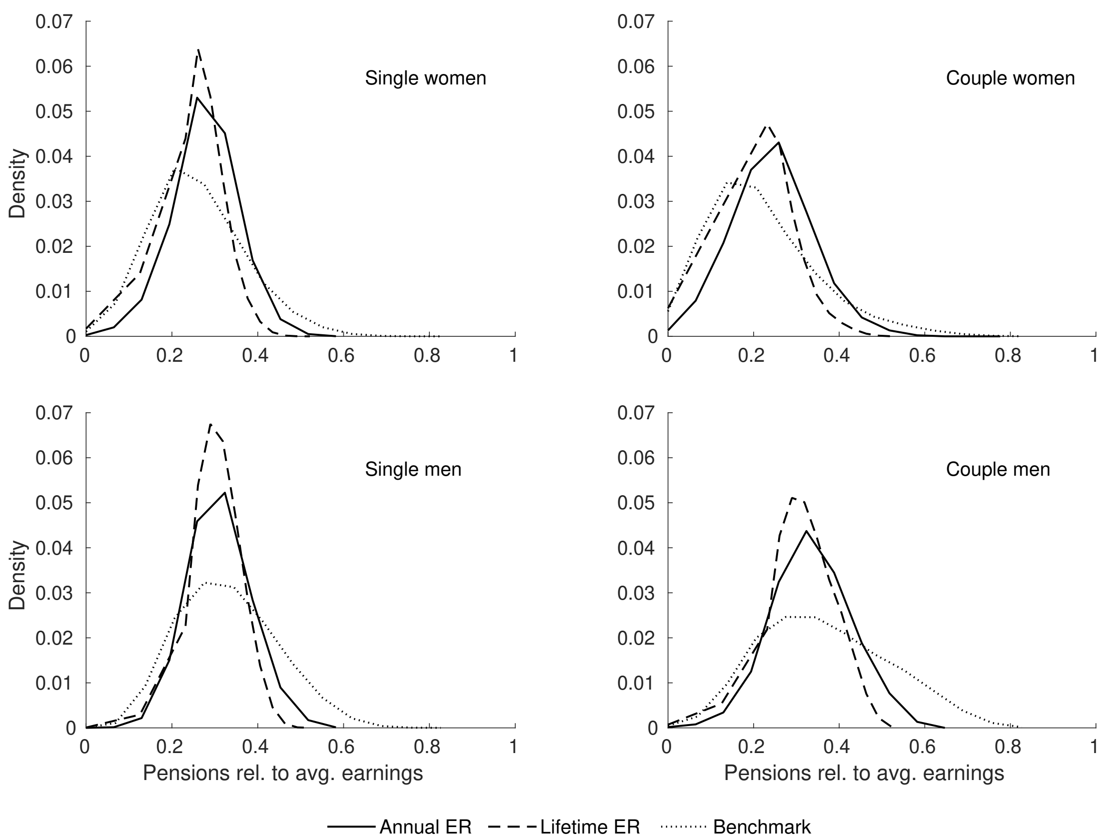
The primary purpose of introducing progressive pension systems into the economy is to reduce old-age income inequality. In this section, we therefore study changes in the distribution of pension income, but also in consumption and wealth.

The distribution of pension income Figure 10 shows the distribution of pension payments at the retirement age j_r relative to average labor earnings $\bar{y}$. The dotted line denotes the initial equilibrium, the solid and the dashed lines the two

[18]This is an individual who exactly earns the average earnings $\bar{y}$ in every period of working life. Hence, the average earner is by construction not directly affected by either of the two reforms, see Figure 1.

reform scenarios, respectively. Both progressive pension reforms narrow the distribution of pension payments relative to a proportional system. However, there are also remarkable differences. The lifetime-earnings-based system compresses the distribution of pension payments the most. Yet, it hardly takes away any of the distribution's left tail, especially for women. The share of women without any pension claims remains the same as in the initial equilibrium. In contrast, the distribution of pension payments is substantially shifted to the right under annual-earnings-based redistribution. Notably, the proportion of both single and married women without any pension payments more than halved. The reason for this is that the annual-earnings-based system stimulates employment especially for workers who would otherwise only work in minijobs or not at all. The additional working years paired with a sizeable earnings subsidy successfully reduce the very left tail of the old-age income distribution. Under a lifetime-earnings-based progressive pension, the very same workers rather drop out of the labor force (either temporarily or permanently) and therefore miss out on important years to accumulate pension payments.

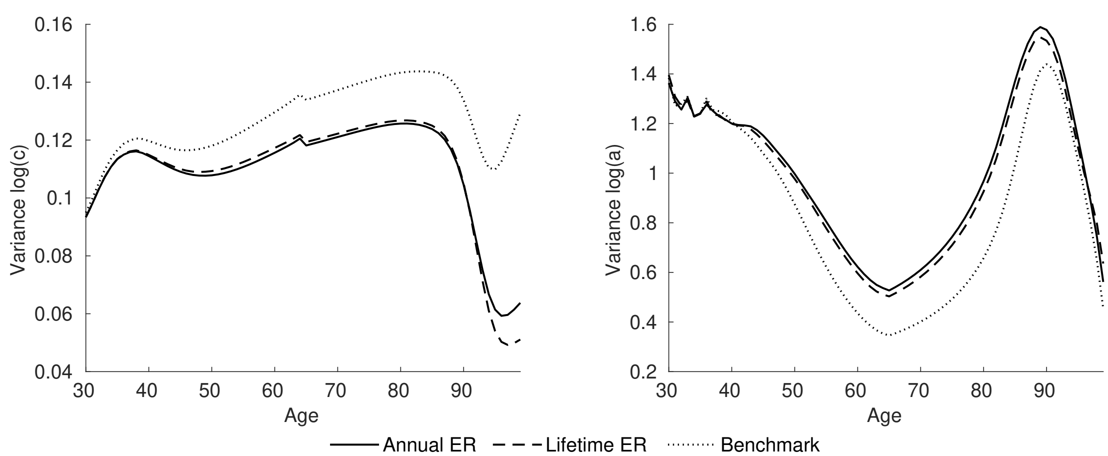
Figure 10: Distribution of pension payments p at retirement entry



Summing up, a lifetime-earnings-based system would in principle have the potential to create a more compressed distribution of pension payments by conditioning redistribution on permanent rather than transitory income. It fails to do so, however, because of its negative employment incentives. In the end, keeping up work turns out to be a better insurance against the risk of old-age poverty than government redistribution.

The variance of consumption and savings Altering the distribution of old-age income also triggers responses in consumption and savings behavior. The left panel of Figure 11 shows the variance of log-consumption over the life cycle. The dotted line again marks the initial equilibrium, while the solid and the dashed lines indicate the two reform scenarios. The variance of consumption decreases markedly for all cohorts as a response to the reforms. Households can smooth the benefits of reduced old-age income inequality over their entire life cycle by adjusting their savings behavior. The drop in consumption inequality is especially pronounced for the very old who have consumed most of their savings and are living solely on pension income. Annual earnings redistribution is more successful in reducing consumption inequality during working life, while lifetime earnings redistribution performs better at very old ages. The annual-earnings-based system pulls households into the workforce during periods of low productivity. This alone reduces gross income inequality and thus consumption inequality at working ages. The situation flips for very old cohorts for which the distribution of pension payments is the only thing that matters. The lifetime-earnings-based system reduces the inequality in pension payments even more than the annual-earnings-based system. Recall, however, that it does so at a lower level of aggregate pension payments.

Figure 11: Variance of consumption log(c) and assets log(a)



The right panel of Figure 11 shows the variance of the log of private wealth over the life cycle. The wealth gap widens substantially as a side effect of the reform. Poor households have to provide less for retirement on their own because of increased insurance through the pension system. The earnings rich, on the other hand, increase their savings rate to compensate for a short-fall in pension income. This increases the variance of wealth starting from age 40 for the remainder of the individual life cycle.

4.7 Welfare Analysis

Reforms towards progressive pension systems come at a more equal distribution of old-age income and at the cost of labor supply distortions and a weaker macroeconomic performance. To paint a more complete picture of their economic impact, we study the effects on ex-ante long-run welfare expressed in consumption equivalent variation CEV.[19] Table 6 shows the effects of both pension reforms for the total population and its subgroups by gender and family status. The column "Total" denotes the entire ex-ante welfare effect, the columns "Cons." and "Labor" decompose the CEV into effects that stem from changes in the utility from consumption and the disutility from labor, respectively.

Table 6: Change in ex-ante long-run welfare

	Annual ER			Lifetime ER		
	Total	Cons.	Labor	Total	Cons.	Labor
Total population	0.28	0.09	0.19	−0.58	−1.22	0.64
– Single women	1.21	0.64	0.56	0.59	−0.82	1.41
– Married women	0.21	−0.10	0.30	−0.82	−1.32	0.50
– Single men	0.10	0.15	−0.04	−0.53	−1.31	0.78
– Married men	−0.11	−0.10	−0.02	−1.08	−1.31	0.23

Welfare effects in CEV (%) over initial equilibrium with proportional pension system.

Annual-earnings-based redistribution Annual-earnings-based redistribution is clearly superior, providing long-run welfare gains of 0.28 percent. The welfare effects result from both a slight increase in the utility from consumption and a decline in the disutility from labor. There are two counteracting forces that shape the utility from consumption: a long-run decline in aggregate consumption depresses the consumption utility, but a reduction in the variance of consumption increases it, see Table 5 and Figure 11. In total, the positive distributional effects

[19] The CEV indicates by how many percent we would have to increase or decrease the consumption level of households at each age and each potential state in the initial equilibrium in order to make them as well off as in a reform scenario with progressive pensions. A positive value for CEV indicates that a reform of the pension system improves long-run welfare and vice versa.

weigh larger than the negative mean effects. The disutility from labor, on the other hand, falls with a lower level of intensive margin labor supply. A higher labor force participation counteracts this effect. Overall, the intensive margin effect is stronger than the extensive margin effect, leading to a lower disutility from labor.

Women, and in particular single women, are the main beneficiaries of such a reform. Owing to increased labor force participation, their old-age provision increases as do their labor earnings. The effects are much stronger for single than for married women, as the latter experience redistribution within the family. This can readily be seen from the changes in utility from consumption. While married women are encouraged to work, their husbands, who typically are primary earners, receive a cut in pension benefits. This internal redistribution of pension payments leads both married men and women to experience a small decline in consumption utility. Single women, on the other hand, purely benefit from redistribution across households, which rationalizes their much higher welfare numbers.

Lifetime-earnings-based redistribution Under lifetime-earnings-based redistribution welfare deteriorates by almost 0.58 percent. The predominant channel at work is a substantial decline in the utility from consumption. This is not surprising given the fact that aggregate consumption declines by a remarkable 2.02 percent in this reform scenario, see Table 5. The distributional properties of the two pension systems only differ slightly. As a result, consumption utility falls unanimously for all population groups. Disutility from labor declines much more than under annual-earnings-based redistribution. This is a direct consequence of the additional extensive margin distortions, which lead to a fall in aggregate employment. Overall, only single women benefit somewhat from this reform scenario, as they obtain additional insurance for which they don't have to provide additional labor effort. Consequently, they experience the smallest decline in consumption utility and the largest fall in disutility from labor.

5 Conclusion

Many real-world pension systems redistribute old-age income based on lifetime earnings. We show that annual-earnings-based redistribution in the pension system would perform significantly better on several dimensions. In particular, it addresses both the causes and the consequence of old-age poverty. The system provides employment incentives to groups that are most prone to ending up with insufficient resources at retirement. This includes, first and foremost, single women and especially single mothers. It also incentivizes labor supply for married women. However, their welfare gains are much smaller than those of single women, as they also benefit from intra-family insurance even under a proportional pension system. The progressive pension system crowds-out some of this insurance by redistributing pension income within the family. The activation of a broader workforce limits the negative macroeconomic consequences of an annual-earnings-based progressive pension reform. On top, it leads to a more favorable distribution of pension claims and also of working age income. Under a lifetime-earnings-based progressive system, women in particular drop out of the labor force and become more dependent on pension subsidies in old age. Under an annual-earnings-based redistribution, individuals need to work in order to enjoy the benefit of redistribution. It turns out that working and enjoying a pension subsidy is much more fruitful in fighting the risk of old-age poverty than waiting for fiscal redistribution, as it improves both the individual and the macroeconomic situation.

Our analysis also comes with some limitations. We only study the welfare effects of pension policy across long-run equilibria, therefore neglecting potential transitional costs and benefits. However, looking at the macroeconomic consequences suggests that we should expect welfare effects along the transition to be even higher. The results in Table 5 show that private savings – a prime indicator for intergenerational redistribution – fall by about the same amount of 1.1 to 1.2 percent in both reform scenarios. This means that we expect intergenerational redistribution to be similar in both reforms and that this redistribution works to the disadvantage of long-run generations. In another study, we confirm this view in a simpler model with a transition path, see Kindermann and Pueschel (2023). In addition to studying a transition, one may want to look at the joint determination of progressivity of the tax and pension system, like in Ábrahám et al. (2023). This, however, goes beyond the scope of this paper a should be left for future research.

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Women's Labor Supply Incentives and Old-Age Income Redistribution

Appendix for Online Publication

Fabian Kindermann and Veronika Püschel

A Building Intuition: Analytical Derivations

Let us assume a household who starts her economic life at age 20 and lives for J years. The household can work until she enters retirement at age j_r. Labor productivity is risky in this model, but survival is certain. This means that the individual doesn't know her exact wage path $\{w_j\}_{j=20}^{j_r-1}$ in advance. The household maximizes the intertemporal utility function

$$U = E_{20}\left[\sum_{j=20}^{J} c_j - \sum_{j=1}^{j_r-1} v(\ell_j)\right],$$

where $\ell_j \in \{0, 1\}$ denotes the labor supply decision at age j. For simplicity, we assume that utility is linear in consumption.[20] $v(\ell)$ indicates the disutility of labor with $v'(\ell) > 0$ and $v''(\ell) > 0$. There is no discounting.

The household earns gross labor income $y_j = w_j\ell_j$. From this income, she has to pay contributions at rate τ_p to the pension system. In reward for her contributions, the individual receives a pension payment at retirement, which is calculated from

$$p = \kappa \times F(y_{20}, y_{21}, \dots, y_{j_r-1}).$$

The intertemporal budget constraint of the household consequently reads

$$\begin{aligned} E_{20}\left[\sum_{j=20}^{J} c_j\right] &= E_{20}\left[\sum_{j=20}^{j_r-1}(1-\tau_p)w_j\ell_j + \sum_{j=j_r}^{J}\kappa \times F(y_{20}, y_{21}, \dots, y_{j_r-1})\right] \\ &= E_{20}\left[\sum_{j=20}^{j_r-1}(1-\tau_p)w_j\ell_j + \kappa \times [J - j_r + 1] \times F(w_{20}\ell_{20}, w_{21}\ell_{21}, \dots, w_{j_r-1}\ell_{j_r-1})\right], \end{aligned}$$

where we assumed (again for simplicity) that the interest rate is equal to $r = 0$.

[20] In our full quantitative simulation model, we relax this assumption and let there be curvature in both utility of consumption and disutility of labor.

Combining the budget constraint and the household utility function, the household's optimization problem reads

$$\max_{\ell_{20},\ell_{21},\dots,\ell_{j_r-1}} \quad E_{20}\left[\sum_{j=20}^{j_r-1}(1-\tau_p)w_j\ell_j+\kappa\times[J-j_r+1]\right.$$
$$\left.\times F(w_{20}\ell_{20},w_{21}\ell_{21},\dots,w_{j_r-1}\ell_{j_r-1})-\sum_{j=20}^{j_r-1}v(\ell_j)\right].$$

If labor supply was a continuous choice variable, the first order condition with respect to labor supply in period j would read

$$v'(\ell_j)=(1-\tau_p)w_j+\kappa\times[J-j_r+1]\times w_j\times E_j\left[\frac{\partial F(y_{20},\dots,w_j\ell_j,\dots,y_{j_r-1})}{\partial(w_j\ell_j)}\right].$$

In a discrete choice framework, where the household can only decide between $\ell_j=0$ and $\ell_j=1$, the discrete version of this equation reads

$$v(1)-v(0)=(1-\tau_p)w_j+\kappa\times[J-j_r+1]$$
$$\times E_j\Big[[F(y_{20},\dots,w_j,\dots,y_{j_r-1})-F(y_{20},\dots,0,\dots,y_{j_r-1})]\Big].$$

It is hence the effective net return to working

$$w_{j,net}=(1-\tau_p)w_j+\kappa\times[J-j_r+1]$$
$$\times E_j\Big[\underbrace{[F(y_{20},\dots,w_j,\dots,y_{j_r-1})-F(y_{20},\dots,0,\dots,y_{j_r-1})]}_{\text{change in earnings record when working at age } j}\Big].$$

that is the prime determinant of the individual's labor force participation decisions.

Proportional system Under a proportional pension system, we have

$$F(y_{20},y_{21},\dots,y_{j_r-1})=\frac{\sum_{j=20}^{j_r-1}y_j}{j_r-20}.$$

Consequently, we get

$$E_j\left[F(y_{20},\dots,w_j,\dots,y_{j_r-1})-F(y_{20},\dots,0,\dots,y_{j_r-1})\right]$$
$$=E_j\left[\sum_{i=20,i\neq j}^{j_r-1}\frac{y_i}{j_r-20}\right]+\frac{w_j}{j_r-20}-E_j\left[\sum_{i=20,i\neq j}^{j_r-1}\frac{y_i}{j_r-20}\right]=\frac{w_j}{j_r-20}.$$

The net return to working at age j under such a system therefore is

$$w_{j,net}^{\mathrm{PR}}=(1-\tau_p)w_j+\kappa\times[J-j_r+1]\times\frac{w_j}{j_r-20}.$$

Lifetime-Earnings-Based Progressive Pensions In a lifetime-earnings-based progressive pension we have

$$F(y_{20}, y_{21}, \dots, y_{j_r-1}) = f\left(\frac{\sum_{j=20}^{j_r-1} y_j}{j_r - 20}\right).$$

Therefore we get

$$\begin{aligned} &E_j\left[F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})\right] \\ &\qquad = E_j\left[f\left(\sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20} + \frac{w_j}{j_r-20}\right) - f\left(\sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20}\right)\right]. \end{aligned}$$

Since $f(\cdot)$ is a (piecewise) linear function, we can immediately write

$$\begin{aligned} &E_j\left[f\left(\sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20} + \frac{w_j}{j_r-20}\right) - f\left(\sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20}\right)\right] \\ &\qquad = E_j\left[f'\left(\frac{\sum_{i=20}^{j_r-1} y_i}{j_r-20}\right)\right] \cdot \frac{w_j}{j_r-20}. \end{aligned}$$

This equation results from a first-order Taylor approximation, which is exact for linear functions. Note that, strictly speaking, this equality doesn't hold in the rare case that

$$\sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20} < b\bar{y} \quad \text{and} \quad \sum_{i=20, i\neq j}^{j_r-1} \frac{y_i}{j_r-20} + \frac{w_j}{j_r-20} > b\bar{y}.$$

But for illustrative purposes, our approximation suffices. The net return to working at a given age j reads

$$w_{j,net}^{\text{LE}} = (1-\tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r-20} \times E_j\left[f'\left(\frac{\sum_{j=20}^{j_r-1} y_j}{j_r-20}\right)\right].$$

Annual-Earnings-Based Progressive Pensions In an annual-earnings-based progressive pension we have

$$F(y_{20}, y_{21}, \dots, y_{j_r-1}) = \frac{\sum_{j=20}^{j_r-1} f(y_j)}{j_r-20}.$$

We consequently have

$$\begin{aligned} &E_j\left[F(y_{20}, \dots, w_j, \dots, y_{j_r-1}) - F(y_{20}, \dots, 0, \dots, y_{j_r-1})\right] \\ &\qquad = E_j\left[\sum_{i=20, i\neq j}^{j_r-1} \frac{f(y_i)}{j_r-20}\right] + \frac{f(w_j)}{j_r-20} - E_j\left[\sum_{i=20, i\neq j}^{j_r-1} \frac{f(y_i)}{j_r-20}\right] = \frac{f(w_j)}{j_r-20} \end{aligned}$$

and the net return to working becomes

$$w_{j,net}^{\text{AE}} = (1-\tau_p)w_j + \kappa \times [J - j_r + 1] \times \frac{w_j}{j_r-20} \times \frac{f(w_j)}{w_j}.$$

B Simulation Model: Computational Details

In this Appendix, we describe further details of the quantitative simulation model. We first show the first order conditions of the household optimization problem. We then describe the stationary recursive competitive equilibrium of the economy and show how to compute the invariant measure of households. Finally, we briefly describe the computational algorithm.

B.1 The Single Household: First Order Conditions

In the following, we derive the solution of the single household's problem in an economy with a proportional pension system. Since labor supply constitutes a discrete choice, we can not formulate a first-order condition for labor supply. Instead, we solve the problem in two steps. First, we assume the household had already made a labor supply choice ℓ. Conditional on this labor supply decision, we determine the optimal consumption-saving decision by solving the conditional optimization problem $\tilde{v}(\mathbf{x_s}, \ell)$.

Consumption–savings choice: The dynamic household optimization problem reads

$$\tilde{v}(\mathbf{x_s}, \ell) = \max_{c_g, a^+} \frac{c_g^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{\left(\zeta_{k,s,g} + \ell\right)^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell>0} + \beta \psi_{j+1,g}^{s} E\left[v(\mathbf{x_s^+}) \middle| j, e, \eta, g, h, k, \ell \right]$$

with $\mathbf{x_s} = (j, g, e, \eta, h, \xi, k, a, d)$ and $\mathbf{x_s^+} = (j+1, g, e, \eta^+, h^+, \xi^+, k^+, a^+, d^+)$. Households maximize their conditional utility with respect to the budget constraint

$$(1+\tau_c) \times c_g \times v(j,k,s) + a^+ + T_p(y, y_{mini}) + T\Big(y, 0, p, s\Big) = (1+r)a + y + y_{mini} + p + t(k,s) + b. \quad (17)$$

The first-order conditions of the household then read

$$\frac{\partial \mathcal{L}}{\partial c} = c_g^{-\frac{1}{\sigma}} - \mu(1+\tau_c)v(j,k,s) = 0$$
$$\frac{\partial \mathcal{L}}{\partial a^+} = -\mu + \beta \psi_{j+1,g}^{s} E\left[v_a(\mathbf{x_s^+}) \;\middle|\; j, e, \eta, g, h, k, \ell \right] = 0,$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. Using the envelope theorem, we immediately obtain

$$v_a(\mathbf{x_s^+}) = (1+r)\mu^+.$$

The Euler equation then reads

$$\frac{c_g^{-\frac{1}{\sigma}}}{(1+\tau_c)\times \upsilon(j,k,s)}$$
$$= (1+r)\beta\psi^s_{j+1,g}E\left[\frac{c_g^+(\mathbf{x_s}^+)^{-\frac{1}{\sigma}}}{(1+\tau_c)\times\upsilon(j+1,k^+,s)}\,\middle|\, j,e,\eta,g,h,k,\ell\right]. \quad (18)$$

The Euler equation (18) and the budget constraint (17) define the optimal level of total household consumption $c(\mathbf{x_s},\ell) = c_g \times \upsilon(j,k,s)$, savings $a^+(\mathbf{x_s},\ell)$ and the utility value $\tilde{v}(\mathbf{x_s},\ell)$ conditional on a certain labor supply decision $\ell \in \{0, \ell_{full}\}$ for single men and $\ell \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$ for female households.[21] Furthermore, the law of motion for lifetime insured earnings

$$d^+(\mathbf{x_s},\ell) = e + y_{ins}(\mathbf{x_s},\ell) \quad (19)$$

determines the conditional lifetime insured earnings $d^+(\mathbf{x_s},\ell)$.

Labour supply decision: Given the utility $\tilde{v}(\mathbf{x_s},\ell)$ for every possible $\ell \leq h$, the utility maximizing labor supply decision is

$$\ell(\mathbf{x_s},\xi) = \arg\max \tilde{v}(\mathbf{x_s},\ell).$$

B.2 The Couples Household: First Order Conditions

The solution of the couple household's problem is quite similar. We first solve the consumption-saving problem conditional on the labor supply choices $\ell_m \in \{0, \ell_{full}\}$ and $\ell_f \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$. For every possible and valid (ℓ_m, ℓ_f) combination we solve the optimization problem

$$\begin{aligned}\tilde{v}(\mathbf{x_c},\ell_m,\ell_f) = \max_{\substack{c_m,c_f,\\ a^+,d_m^+,d_f^+}} &\left[\frac{c_m^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_m\frac{(\zeta_{k,c,m}+\ell_m)^{1+\frac{1}{\chi_m}}}{1+\frac{1}{\chi_m}} - \xi\times\mathbb{1}_{\ell_m>0}\right.\\ &\left. + \frac{c_f^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_f\frac{(\zeta_{k,c,f}+\ell_f)^{1+\frac{1}{\chi_f}}}{1+\frac{1}{\chi_f}} - \xi\times\mathbb{1}_{\ell_f>0}\right]\\ &+ \beta\psi^c_{j+1}E\Big[v(\mathbf{x_c}^+)\,\Big|\, j,e_m,e_f,\eta_m,\eta_f,h,k,\ell_f\Big]\end{aligned}$$

with

$$\mathbf{x_c} = (j, e_m, e_f, \eta_m, \eta_f, h, \xi, k, a, d_m, d_f) \quad \text{and}$$
$$\mathbf{x_c}^+ = (j+1, e_m, e_f, \eta_m^+, \eta_f^+, h^+, \xi^+, k^+, a^+, d_m^+, d_f^+).$$

[21] Note that the female household might be constraint in her labor choice set, i.e. $\ell \leq h$ must hold.

Couples maximize their utility subject to the borrowing constraint $a^+ \geq 0$, and the budget constraint

$$\begin{aligned}(1+\tau_c) \times (c_m + c_f) \times \upsilon(j,k,c) + a^+ + T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big) \\ + T\Big(y_m, y_f, p, c\Big) = (1+r)a + y + y_{mini} + p + t(k,c) + b. \quad (20)\end{aligned}$$

The first-order conditions read

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_g} &= c_g^{-\frac{1}{\sigma}} - \mu(1+\tau_c)\upsilon(j,k,c) = 0 \\ \frac{\partial \mathcal{L}}{\partial a^+} &= -\mu + \beta\psi_{j+1}^c E\left[\tilde{v}_a(\mathbf{x_c}, \ell_m, \ell_f) \;\middle|\; j, e_m, e_f, \eta_m, \eta_f, h, k, \ell_f\right] = 0,\end{aligned}$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. We immediately see that $c_m = c_f$ needs to hold. Using the envelope theorem, we obtain

$$v_a(\mathbf{x_c^+}) = (1+r)\mu^+.$$

The Euler equation then reads

$$\begin{aligned}&\frac{c_g^{-\frac{1}{\sigma}}}{(1+\tau_c) \times \upsilon(j,k,c)} \\ &\quad = (1+r)\beta\psi_{j+1}^c E\left[\frac{c_g^{-\frac{1}{\sigma}}(\mathbf{x_c^+})}{(1+\tau_c) \times \upsilon(j+1,k^+,c)} \middle|\; j, e_m, e_f, \eta_m, \eta_f, h, k, \ell_f\right]. \quad (21)\end{aligned}$$

The Euler equation (21) and the budget constraint (20) define the optimal level of total household consumption $c(\mathbf{x_c}, \ell_m, \ell_f) = (c_m + c_f) \times \upsilon(j,k,c)$, savings $a^+(\mathbf{x_c}, \ell_m, \ell_f)$ and the utility value $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ conditional on a certain labor supply decision (ℓ_m, ℓ_f). Furthermore, the laws of motion for lifetime insured earnings

$$d_g^+(\mathbf{x_c}, \ell_g) = d_g + y_{ins,g}(\mathbf{x_c}, \ell_g) \quad (22)$$

determines the conditional lifetime insured earnings for each partner $d_g^+(\mathbf{x_c}, \ell_g)$.

Labour supply decision: Given the conditional utility $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ for every possible (ℓ_m, ℓ_f) combination with $l_f \leq h$, the household chooses the utility maximizing labor supply combination

$$\Big(\ell_m(\mathbf{x_c}), \ell_f(\mathbf{x_c})\Big) = \arg\max \tilde{v}(\mathbf{x_c}, \ell_m, \ell_f).$$

B.3 Stationary Recursive Competitive Equilibrium

Definition 1. *Given an international interest rate $\bar{r}$, government expenditures G, a consumption tax rate τ_c, a progressive tax system $T(\cdot)$ as well as a characterization of the pension system $\{\tau_p, \varrho, \kappa\}$, a stationary recursive equilibrium is a collection of value and policy functions $\{v, c, \ell, a^+, d^+\}$ for a single and $\{v, c, \ell_m, \ell_f, a^+, d_m^+, d_f^+\}$ for a couple household, optimal production inputs $\{K, L\}$, accidental bequests $\{b_j\}_{j=1}^J$, a net foreign asset position and a trade balance $\{Q, TB\}$ as well as factor prices $\{r, w\}$ that satisfy*

1. *(Household Optimization) Given prices and characteristics of the tax and pension system, the value function v satisfies the Bellman equation* (9) *for singles and* (10) *for couples together with the budget constraint* (8)*, the accumulation equations for pension claims* (13) *and* (14)*, the borrowing constraint $a^+ \geq 0$ and the laws of motion for productivity risk, career choice and fertility. c, ℓ, a^+ and d^+ are the associated policy functions for singles and $c, \ell_m, \ell_f, a^+, d_m^+, d_f^+$ are the policy functions for couples.*

2. *(Firm Optimization) Given the international interest rate $\bar{r}$ as well as the wage rate w, firms employ capital and labor according to the demand functions*

$$\bar{r} = \Omega\alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta \;\; \textit{and} \;\; w = \Omega(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

Aggregate output is calculated from (11) *and the capital stock evolves according to* (12)*.*

3. *(Government Constraints) The budget constraint of the pension system*

$$\underbrace{\int p(d) \times \mathbb{1}_{j \geq j_r} \; d\Phi^s + \int \Big[p(d_m) + p(d_f)\Big] \times \mathbb{1}_{j \geq j_r} \; d\Phi^c}_{\textit{total pension claims}} =$$
$$\underbrace{\int T_p\Big(y_g, y_{mini,g}\Big) \; d\Phi^s + \int T_p\Big(y_m, y_{mini,m}\Big) + T_p\Big(y_f, y_{mini,f}\Big) \; d\Phi^c}_{\textit{total contributions}}. \quad (23)$$

and that of the tax system

$$\tau_c \times C + \int T\Big(y_g, 0, p, s\Big) \; d\Phi^s + \int T\Big(y_m, y_f, p, c\Big) \; d\Phi^c$$
$$= G + \int t(k, s) \; d\Phi^s + \int t(k, c) \; d\Phi^c \quad (24)$$

hold. Accidental bequests are calculated from

$$b_j = \frac{\int \frac{1-\psi^s_{j,g}}{\psi^s_{j,g}} \times (1+r)a \; d\Phi^s + \int \frac{1-\psi^c_j}{\psi^c_j} \times (1+r)a \; d\Phi^c}{\int \mathbb{1}_{j<j_R} \; d\Phi^s + \int \mathbb{1}_{j<j_R} \; d\Phi^c} \quad \textit{if } j < j_R. \quad (25)$$

4. *(Market Clearing:)*

   (a) *The labor market clears:*

   $$L = \int w(j, e, \eta, g, k, \ell) l(\boldsymbol{x}_s) \; d\Phi^s + \int w(j, e, \eta, g, k, \ell) l(\boldsymbol{x}_c) \; d\Phi^c$$

   Note, we let the wage w depend on ℓ due to the fixed wage rate in minijobs.

   (b) *The capital market clears:*

   $$K + Q = \int a \; d\Phi^s + \int a \; d\Phi^c$$

   (c) *The balance of payments identity is satisfied:*

   $$TB = -\bar{r}Q$$

   (d) *The goods market clears:*

   $$Y = \int c(\boldsymbol{x}_s) \; d\Phi^s + \int c(\boldsymbol{x}_c) \; d\Phi^c + \delta K + G + TB.$$

5. *(Consistency of Probability Measure Φ) The invariant probability measure is consistent with the population structure of the economy, with the exogenous processes of labor productivity η, labor flexibility h and fertility k, and the household policy functions a^+ and d^+ or d_m^+ and d_f^+, respectively. A formal definition of the probability measures is provided in Appendix B.4.*

B.4 The Measure of Households

The population consists of couple and single households which operate on different state spaces. At age 20, the mass of couple households Φ^c and the mass of single households Φ^s sum to one.

Couple households At age 20, couple households draws one of four possible education level (e_m, e_f) from the joint distribution $\phi_e^c(e_m, e_f)$. Conditional on the their education level, each partner draws an initial labor productivity η from the invariant distribution $\pi_{\eta,20}$ of the process for η. The household enters the economy without kids $k = 0$, the full labor choice set $h = \ell_{full}$, zero assets $a = 0$ and with zero insured lifetime earnings $d_m = d_f = 0$. The realization of ξ follows a log-normal distribution $\pi_\xi(\xi)$ with mean μ_ξ and variance σ_ξ^2. Thus, the measure of couple households with characteristics $\mathbf{x_c} = (e_m, e_f, \eta_m, \eta_f, h, \xi, k, a, d_m, d_f)$ is constructed as

$$\Phi^c(\{20\}, \{e_m\}, \{e_f\}, \{\eta_m\}, \{\eta_f\}, \{\ell_{full}\}, \{\xi\}, \{0\}, \{0\}, \{0\}, \{0\}) = \\ \phi_c \times \phi_e^c(e_m, e_f) \times \pi_{\eta_m,20}(\eta_m \,|\, e_m) \times \pi_{\eta_f,20}(\eta_f \,|\, e_f) \times \pi_\xi(\xi),$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and insured lifetime earnings of the husband $\mathcal{D}_m$ and the wife $\mathcal{D}_f$ we have

$$\begin{aligned}
&\Phi^c(\{j+1\},\{e_m\},\{e_f\},\{\eta_m^+\},\{\eta_f^+\},\{h^+\},\{\xi^+\},\{k^+\},\mathcal{A},\mathcal{D}_m,\mathcal{D}_f) = \\
&= \psi_{j+1}^c \times \pi_\eta(\eta_m^+ \mid \eta_m, e_m) \times \pi_\eta(\eta_f^+ \mid \eta_f, e_f) \times \pi_h(h^+ \mid h, f, \ell_f) \times \pi_\xi(\xi) \times \pi_k(k^+ \mid k, j, c, f) \\
&\qquad \times \int \mathbb{1}_{\{a^+(\mathbf{x_c})\in\mathcal{A}\}} \times \mathbb{1}_{\{d_m^+(\mathbf{x_c})\in\mathcal{D}_m\}} \times \mathbb{1}_{\{d_f^+(\mathbf{x_c})\in\mathcal{D}_f\}} \\
&\qquad\qquad \Phi^c(\{j\},\{e_m\},\{e_f\},\{\eta_m\},\{\eta_f\},\{h\},\{\xi\},\{k\},da,dd_m,dd_f),
\end{aligned}$$

where the integral is the measure of assets a and lifetime insured earnings d_m and d_f today such that for fixed $(j, e_m, e_f, \eta_m, \eta_f, h, \xi, k, \ell_f)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_c})$ lies in $\mathcal{A}$ and the optimal choice today of lifetime insured earnings for tomorrow $d_m^+(\mathbf{x_c})$ and $d_f^+(\mathbf{x_c})$ lie in $\mathcal{D}_m$ and $\mathcal{D}_f$, respectively.

Single households Next, we construct the measure of single households across the characteristics $\mathbf{x_s} = (g, e, \eta, h, \xi, k, a, d)$. At age 20, households draw a gender $g \in \{0, 1\}$ and an education level $e \in \{0, 1\}$, where $g = 1$ occurs with probability ϕ_g and $e = 1$ with probability $\phi_e^{s,g}$. Conditional on the education level, households draw an initial labor productivity η from the distribution $\pi_{\eta,20}$, see above. Households enter the economy without kids, the full labor choice set $h = \ell_{full}$, zero assets and zero lifetime insured earnings. Thus,

$$\begin{aligned}
&\Phi^s(\{20\},\{g\},\{e\},\{\eta\},\{\ell_{full}\},\{\xi\},\{0\},\{0\},\{0\}) \\
&\qquad\qquad\qquad = \phi_s \times \phi_g(g) \times \phi_e^{s,g}(e) \times \pi_{\eta,20}(\eta \,|\, e) \times \pi_\xi(\xi),
\end{aligned}$$

and zero otherwise.

We can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and lifetime insured earnings $\mathcal{D}$ we have

$$\begin{aligned}
&\Phi(\{j+1\},\{g\},\{e\},\{\eta^+\},\{h^+\},\{\xi^+\},\{k^+\},\mathcal{A},\mathcal{D}) = \\
&= \psi_{j+1,g}^s \times \pi_\eta(\eta^+ \mid \eta, e) \times \pi_h(h^+ \mid h, g, \ell) \times \pi_\xi(\xi) \times \pi_k(k^+ \mid k, j, s, g) \\
&\qquad \times \int \mathbb{1}_{\{a^+(\mathbf{x_s})\in\mathcal{A}\}} \times \mathbb{1}_{\{d^+(\mathbf{x_s})\in\mathcal{D}\}}\ \Phi(\{j\},\{s\},\{\eta\},\{h\},\{\xi\},\{k\},da,dd)
\end{aligned}$$

where the integral is the measure of assets a and lifetime insured earnings d today such that, for fixed $(j, g, e, \eta, h, \xi, k, \ell)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_s})$ lies in $\mathcal{A}$ and the optimal choice today of lifetime insured earnings for tomorrow $e^+(\mathbf{x_s})$ lies in $\mathcal{D}$.

B.5 Computational Algorithm

Following Kindermann et al. (2020), we solve the model in two steps. First, we apply the method of endogenous grid points to solve the household's consumption-savings problem. We can then compute policy functions $c(\mathbf{x_s}), \ell(\mathbf{x_s}), a^+(\mathbf{x_s})$ for single households and $c(\mathbf{x_c}), \ell_m(\mathbf{x_c}), \ell_f(\mathbf{x_c}), a^+(\mathbf{x_c})$ for couple households as well as the value functions $v(\mathbf{x_s})$ and $v(\mathbf{x_c})$. Second, we determine equilibrium quantities and prices using a standard rootfinding method.

B.5.1 Computation of Policy and Value Functions

This section presents the method for computing the policy and value functions of single households using the method of endogenous gridpoints. The solution method for couple households is technically identical. The state of a single household is given by $\mathbf{x_s} = (j, g, e, \eta, h, \xi, k, a, d)$. To solve the model on a computer, we start with discretizing the continuous elements a, d, η. We use routines provided by the toolbox that accompanies Fehr and Kindermann (2018).

- We specify the asset grid $\hat{\mathcal{A}} = \{\hat{a}_0, \ldots, \hat{a}_{40}\}$ as nodes with growing distance on the interval $[\bar{a}_l, \bar{a}_u]$. In particular, we let

$$\hat{a}_{ii} = \bar{a}_l + \frac{\bar{a}_u - \bar{a}_l}{(1+g_a)^{40} - 1} \times [(1+g_a)^{ii} - 1] \text{ for } ii = 0, 1, \ldots, 40.$$

  The lower limit of the asset grid is $\bar{a}_l = 0$, the upper limit of the asset grid is $\bar{a}_u = 50$, the growth rate of gridpoints is $g_a = 0.14$.

- We specify the grid for insured lifetime earrings $\widehat{\mathcal{D}} = \{\hat{d}_0, \ldots, \hat{d}_{12}\}$ as a grid on the interval $[0, 2]$ with equally spaced nodes.

- We approximate the stochastic process of the AR(1) labor productivity process η by a discrete Markov chain. We use the Rouwenhorst method to discretize the stochastic process $\hat{\mathcal{E}} = \{\hat{\eta}_1, \ldots, \hat{\eta}_5\}$ and to determine a transition matrix

$$\pi_\eta(\eta^+|\eta) = \begin{bmatrix} \pi_{11} & \pi_{12} & \ldots & \pi_{15} \\ \pi_{21} & \pi_{22} & \ldots & \pi_{25} \\ \vdots & \vdots & \ddots & \ldots \\ \pi_{51} & \pi_{52} & \ldots & \pi_{55} \end{bmatrix}. \tag{26}$$

The policy and value functions can now be solved via backward induction. In the last possible age J, the household will not work[22] and not save, but will consume all remaining resources. This determines the policy functions

$$\begin{aligned} c(J, e, \hat{\eta}_g, h, \xi, k, \hat{a}_{ii}, \hat{d}_k) &= \frac{(1+r)\hat{a}^{ii} + p(\hat{d}_k) - T(0, 0, \hat{d}_k, s) + b}{(1+\tau_c) \times v(J, k, s)}, \\ l(J, e, \hat{\eta}_g, h, \xi, k, \hat{a}_{ii}, \hat{d}_k) &= 0, \\ a^+(J, e, \hat{\eta}_g, h, \xi, k, \hat{a}_{ii}, \hat{d}_k) &= 0 \end{aligned}$$

[22]Remember, the compulsory retirement age is j_r.

and the value function

$$v(J, e, \hat{\eta}_g, h, \xi, k, \hat{a}_{ii}, \hat{d}_k) = \frac{c(J, e, \hat{\eta}_g, h, \xi, k, \hat{a}_{ii}, \hat{d}_k)^{1-\sigma}}{1-\sigma}$$

for all $g = 0, \ldots, 5,\ ii = 0, \ldots, 40,\ k = 0, \ldots, 12.$

With the final period policy functions and value function at hand, we can iterate backwards over ages to determine the full history of household decisions. We now only describe the procedure for working-age single households. The optimization problem as well as the first-order conditions for these households are discussed in Appendix B.1.

We now apply the method of endogenous gridpoints. We use the exogenous grid $\hat{\mathcal{A}} = \{\hat{a}_{ii}\}_{ii=0}^{40}$ to indicate the remainder of assets into the next period, i.e. $a^+ = \hat{a}_v$. For each state $\tilde{x} = (j, e, \eta, h, k, \mathbf{a}^+, d)$ and possible labor choice $\ell \in \{0, \ell_{mini}, \ell_{part}, \ell_{full}\}$:

1. we first determine
$$d^+ = \frac{(j-1)d}{j} + \frac{y_{ins}}{j}$$
2. given a^+ and d^+ we determine $c(\tilde{x})$ from the Euler Equation (18)
3. with $l(\tilde{x})$ and $c(\tilde{x})$, we use the budget constraint (17) to get $a(\tilde{x})$.

Once $l(\tilde{x})$, $c(\tilde{x})$ and $a(\tilde{x})$ are solved, we can interpolate along a to obtain the policy functions $l(\mathbf{x_s})$, $c(\mathbf{x_s})$ and $a^+(\mathbf{x_s})$ as well as the value function $v(\mathbf{x_s})$ for each today's asset value $\hat{a}_{ii},\ ii = 0, \ldots, 40$ and insured lifetime earnings $\hat{d}_k, k = 0, \ldots, 12$ by piecewise linear interpolation.

In case the asset restriction $a^+ \geq 0$ is binding, we extend the interpolation data by another point of value 0 on the left and determine the policy and value functions at this point. We assume the household consumes all available resources and has no savings left over for tomorrow.

B.5.2 A Long-Run Equilibrium of the Macroeconomy

We model a small open economy, hence prices r and w are fixed. In order to determine aggregate quantities and policy parameters in the initial equilibrium ($t = 0$) we need to determine the following four variables numerically:

- the government budget balancing consumption tax rate τ_c that satisfies equation (24)
- the pension replacement rate κ that balances pension contributions and pension payments as outlined in equation (23)

- average earnings $\bar{y}_t$ of the employed population[23]
- aggregate bequests $\bar{B}$, which immediately allows us to compute cohort bequests $\{b_j\}_{j=1}^{J}$, see equation (25).

Once a guess of these four variables is available, we can use the following algorithm to compute the remainder individual and aggregate variables of the economy:

1. We solve the household optimization problem using the guesses for $\tau_c, \kappa, \bar{y}, \bar{B}$ and determine the measure of households.
2. We compute aggregate quantities $\{L, K, Q, A, TB, Y, C, G, I, \Omega, B\}$ from individual decisions and the measure of household and determine the gap $D = Y - C - I - G$ between demand and supply.

We determine the four central parameters $(\tau_c, \kappa, \bar{y}_t, \bar{B})$ by means of a quasi-Newton rootfinding method. The method receives an initial guess of these variables and updates them in each iteration step using the (numerical) Jacobian of the determining equation system. The iteration process stops when the government and the pension budget are in equilibrium and the model implied average earning and aggregate bequest equal the guess provided by the method.

[23] This is an important parameter, as it determines the pension contribution cap, pension payments and earnings of the low-earning group.

C The Calibration Process in Detail

This appendix discusses our choices of functional forms and parameters in detail. We pay particular attention to demographics and the labor market characteristics of different types of households. We parameterize the model using data and calibration targets from the German economy, which currently features a proportional pension system. Germany therefore serves as a good benchmark for reforms that aim at introducing progressivity into the pension formula.[24] Our base year is 2017, in which the average earnings – the empirical counterpart to average earnings $\bar{y}$ in our model – amounted to EUR 37,000, see DRV Bund (2020). Our parameterization process is a two-step procedure: We try to identify as many parameters in the model as possible to which we can directly assign values either by estimating them from data or by using direct estimates from the literature. We then use the remaining set of parameters to calibrate the model to data moments, which we again derived from micro and macro data of the German economy. We provide a summary table of all parameters at the end of this appendix.

C.1 Demographics

We let households start their economic life at the age of 20 and allow for a maximum life span of 99 years. The mandatory retirement age j_r is 64, which equals the average retirement age of the German regular retirement population in 2017, see DRV Bund (2019). We use data from the age cohorts 35-49 of the 2017 German Microcensus[25] to estimate the following demographic parameters:

1. 50.78% of individuals in the sample are male, 33.06% of them have a college education, and 67.54% of them live in a couple household.[26]
2. The proportion of women is 49.22% and 27.76% of them have a college education.
3. We also examine patterns of assortative mating and find that 85.69% of non-college educated men are married to a non-college educated woman. 54.81% of college-educated men are married to a college-educated woman.

Informed by these empirical facts, we set the demographic parameters of the model as follows:

1. A share $\phi_m = 0.5078$ of each new-born cohort are men and $\phi_{c,m} = 0.6754$ of them are married, the remainder is single. For women, these shares are $\phi_f = 0.4922$ and $\phi_{c,f} = 0.69680$.

[24]U.S. Social Security would not make the best possible starting point, as it redistributes heavily towards single-earner married couples through spousal and survivor benefits, see for example Kaygusuz (2015). The German system has survivor benefits, too, but no spousal transfers.

[25]See RDC of the Federal Statistical Office and Statistical Offices of the Federal States (2021)

[26]This includes individuals who live in a couple household but are not formally married.

2. The education distribution within couples is given by

$$\phi_e^c(e_m, e_f) = \begin{bmatrix} \phi_e^c(0,0) = 0.5736 & \phi_e^c(0,1) = 0.0958 \\ \phi_e^c(1,0) = 0.1494 & \phi_e^c(1,1) = 0.1812 \end{bmatrix}, \tag{27}$$

where $\phi_e^c(0,0)$ indicates that both partners are non-college educated and $\phi_e^c(1,1)$ that both are college educated. A single woman is college-educated with likelihood $\phi_e^{s,f} = 0.2790$ and a single men with likelihood $\phi_e^{s,m} = 0.3306$.

This calibration strategy provides us with a model consistent measure of households that matches the empirical targets.

C.2 The Structure of the Labor Market

A crucial element for the analysis of any fiscal reform – and pension reforms in particular – is a model that can replicate empirical labor supply patterns across demographic groups and over the life cycle. Labor supply choices are to a large degree determined by labor productivity and preferences. To discipline our choices of the various model parameters that can impact individual labor supply decisions, we proceed as follows: We first estimate average labor productivity profiles and risk processes for men from administrative data of the German pension insurance system. These estimates deliver the general productivity profiles $z(j, e, \eta)$ for workers at different ages j, education levels e and productivity shocks η. We assume that women share the general productivity process with men, but on top they are exposed to a gender wage gape w_{gap} as well as a motherhood wage penalty $w_{pnty}(k)$, which we estimate from data by Schrenker and Zucco (2020).

After all these model parameters are set from direct empirical estimates, we calibrate the remaining parameters of the model so as to tackle empirical statistics on labor supply from the German 2017 Microcensus shown in Table C.1. The data show quite distinct patterns of labor supply across genders, age groups and family status. Labor supply at both the extensive and the intensive margin is, for example, much smaller for older men and women as well as for women that live in a marriage. Single women, on the other hand, work on a full-time job more often than married women. While of course, in a non-linear model, every parameter causes changes in several variables, we will try to identify the data moments onto which a parameter or a set of parameters has the most direct impact. At the end of this appendix, we provide a detailed table of exogenously set and endogenously calibrated parameters as well as their data sources or calibration targets, respectively.

C.2.1 Labor Hours Choices

Labor supply ℓ is modeled as discrete choice, see the discussion in Section 3.2. According to the German Microcensus, full-time employees of ages 25-63 work

Table C.1: Benchmark: labor supply

	Women				Men	
	not empl	minijob	PT	FT	not empl	FT
	Young: Ages 25-44					
Data	27.86	5.94	26.09	40.11	13.30	86.70
Model	23.16	6.22	23.98	46.64	12.45	87.55
	Older: Ages 45-63					
Data	25.09	8.47	31.95	34.50	19.64	80.36
Model	25.53	7.34	33.61	33.52	19.85	80.15
	Singles: Ages 25-63					
Data	22.98	4.03	21.66	51.33	23.73	76.27
Model	22.60	0.01	17.21	60.19	19.34	80.66
	Couples: Ages 25-63					
Data	27.71	8.73	32.55	31.01	12.73	87.27
Model	25.04	9.72	33.58	31.67	14.38	85.62

Data Source: German Microcensus (2017), own calculations.

an average of 40.3 hours per week, whereas part-time employees work 21.0 hours per week. Assuming a maximum time endowment of 100 hours per week (the equivalent to $\ell = 1$) for an individual,[27] we set $\ell_{full} = 0.403$ and $\ell_{part} = 0.210$. For minijobs, we finally set $\ell_{mini} = 0.100$ paying tribute to the fact that those jobs are typically low-hours marginal types of employment.

C.2.2 General Labor Productivity

All individuals of an education level e share a common deterministic age-specific labor productivity profile $\theta_{j,e}$. We estimate these profiles using administrative data from the German public pension insurance (Deutsche Rentenversicherung). The dataset contains detailed information on the monthly history of pension claims earned by each individual as well as the sources these claim were derived from. Pension claims are in general a good indicator for estimating earnings processes, as they are proportional to individual earnings. See Appendix D for details on the data.

Using these data, we estimate the following flexible functional form for the education-

[27] Out of a total of 168 hours per week this means that 49 are reserved for sleeping and another 19 for eating, hygiene, etc.

specific age-productivity profiles:

$$\theta_{j,e} = b_{0,e} + b_{1,e}\frac{\min(j, j_{M,e})}{10} + b_{2,e}\left[\frac{\min(j, j_{M,e})}{10}\right]^2 + b_{3,e}\left[\frac{\min(j, j_{M,e})}{10}\right]^3. \qquad (28)$$

This form is flexible enough to capture both a hump-shaped ($j_{M,e} = \infty$) and a stagnating ($j_{M,e} < j_R$) life-cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_{M,e}$ onward. The upper part of Table C.2 shows the estimation results, the left panel of Figure 4 visualizes the estimated labor productivity profiles as well as their empirical counterpart.

Table C.2: Parameter values of labor productivity profiles and risk

	High school $e = 0$	College $e = 1$
Intercept $b_{0,e}$	-2.0732	-17.2099
Linear age term $b_{1,e}$	0.7833	11.8163
Quadratic age term $b_{2,e}$	-0.0572	-2.6345
Cubic age term $b_{3,e}$	-0.0026	0.1984
Stagnation threshold $j_{M,e}$	∞	44.26
Autocorrelation ρ_e	0.9300	0.9900
Innovation variance $\sigma^2_{\varepsilon,e}$	0.0372	0.0059
Unconditional variance $\frac{\hat{\sigma}^2_{\varepsilon,e}}{1-\hat{\rho}^2_e}$	0.2756	0.2983

Taking residual earnings, we estimate the education-specific parameters of an AR(1)-process for log-productivity as specified in (6). The results are shown in the lower panel of Table C.2. The life-cycle average labor productivity profiles exhibit the usual shape, where earnings rise over time and especially so for college graduates. The processes for labor productivity risk are highly persistent, with a somewhat smaller persistence for high-school workers and a larger persistence for college graduates. The overall unconditional process variance ranges at around 28 to 30 log-points.

According to the German tax law the earnings-threshold for minijobs is 450 Euros per month. Since not every minijob worker earns the maximum amount, we assume minijob earnings of 400 Euros per month that corresponds to 4800 Euros annually or $\bar{y}_{mini} = 0.1297 \times \bar{y}$.

C.3 Preferences and the Budget Constraint

Utility is additively separable in consumption $c_{j,g}$ and labor supply $\ell_{j,g}$, see again (7). Utility from consumption features constant absolute risk aversion σ, utility from labor a constant but gender-specific Frisch elasticity χ_g. On top of disutility from working, parents have to bear the time costs of caring for children amounting to $\zeta_{k,i,g}$, see the discussion below. Finally, participation in the labor market is costly to individuals. Specifically, when choosing labor hours greater than zero, a worker has to pay the participation utility cost ξ. We assume that ξ is drawn at the household-level – meaning that it is common to married couples – but iid across households and across time and independent of individual labor productivity. We let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2.

Preference parameters We assign a value of 2.0 to risk aversion σ, a choice quite typical for the heterogeneous agent macroeconomics literature though at the lower end of values that generate an extensive desire for redistribution.[28] The empirical literature has pointed to the fact that Frisch elasticities differ significantly between men and women, see for example Keane (2011). Consistent with this evidence, we chose values of $\chi_m = 0.4$ for men and $\chi_w = 0.75$ for women. After making these data-based choices, we are left with the set $(\beta, \nu_m, \nu_f, \mu_\xi, \sigma_\xi^2)$ of parameters that we need to calibrate. We choose the time discount factor $\beta = 0.9785$ so that all capital is entirely absorbed by private savings in the initial equilibrium, and net foreign assets as well as the trade balance are zero. We then jointly calibrate $\nu_m = 70.0$, $\mu_\xi = 1.65$ and $\sigma_\xi^2 = 2.5$ to match the participation rates of men across demographic groups in Table C.1. Finally we set $\nu_f = 22.0$ to achieve an overall good divide between minijob, part-time and full-time work for women.

Time costs of children Children need to be taken care of, which we model as time costs when they are present in the household $k \in \{0, 1, 2, 3\}$. Single mothers have to bear the entire time cost of children on their own. We hence calibrate the time cost of young children $\zeta_{1,s,w} = 1.5$ and older children $\zeta_{2,s,w} = 0.25$ to match the labor supply patterns of both single women as well as young women in Table C.1. Married mother exhibit a somewhat different labor supply pattern over the life cycle, which might be either due to the fact that they live in a two earner household or may result from partners partially sharing the cost of raising children. In order to match the empirical labor supply profiles of married women, too, we assume that the overall time costs of raising children are the same in single and married couple families, but that fathers take a certain (small) share of these costs. This leads us to $\zeta_{1,c,w} = 1.20$ and $\zeta_{1,c,m} = 0.30$ for young children as well as $\zeta_{2,c,w} = 0.1875$ and $\zeta_{2,c,m} = 0.0625$ for older children.

[28] In this model, σ fulfils two roles as it defines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter typically range between values of 1 and 3, whereas risk aversion can be quite high and well beyond values of 10 when estimated from individual financial choices, see for example Vissing-Jørgensen and Attanasio (2003).

Labor hours flexibility Finally, we pay tribute to the fact that many mothers continue to work part-time, even when their children have already left the household. Our model would not be able to adequately replicate this pattern without restrictions on mother's flexibility in choosing labor hours. The state $h \in \{\ell_{part}, \ell_{full}\}$ refers to a woman's choice set for labor hours. The transition matrix $\pi_h(h^+|h, g, \ell)$ governs the transition between these states and is conditional on the current labor hours choice ℓ. For women who work full time, we assume that the transition matrix is the identity matrix, meaning that they will not be at the risk of facing labor hours restrictions in the next period. Women who do not work full-time transition from $h = \ell_{full}$ into the state $h = \ell_{part}$ with a likelihood of 0.95. Once in this state, they come back to $h = \ell_{full}$ with an annual probability of 0.15. The average duration of a period of labor hours inflexibility is therefore 6.67 years.

C.4 Technology

We choose a depreciation rate of $\delta = 0.07$, which leads to a realistic investment to output ratio of 21%, see German Statistical Office (2020). We set the capital share in production at $\alpha = 0.3$ to obtain a capital-to-output ratio of three and normalize the technology level Ω such that the wage rate per efficiency unit of labor w is equal to 1. Finally, we assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

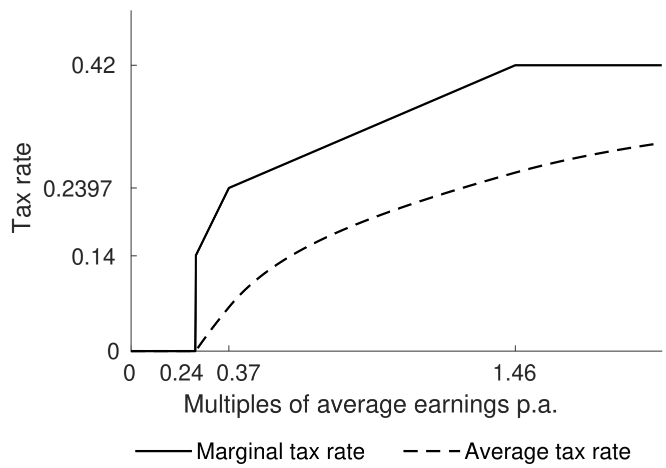
C.5 The Public Pension System

The pension system runs on a pay-as-you-go basis. This means that in equilibrium, total annual pension contributions need to be equal to the total amount of annual pension payments. We fix the pension contribution rate at its statutory rate of $\tau_p = 0.187$ in 2017. The accrual rate for minijob earnings is $\varrho = 0.80$. These choices result in a gross pension replacement of $\kappa = 0.41$, which is similar to the German gross replacement rate for the mean earner as reported by OECD (2021).

C.6 The Tax System and Government Expenditure

The government raises proportional taxes on consumption and progressive taxes on labor earnings to cover government expenditures. We employ the 2017 statutory German progressive income tax code as depicted in Figure C.1 . Recall that couples enjoy a tax advantage in the form of income splitting, see (16). We set the proportional consumption tax rate at $\tau_c = 0.16$ to balance the fiscal budget. This untargeted tax rate is in line with the German VAT tax. Although consumption goods are generally taxed at a rate of 19%, many goods (such as food, books and newspapers) are taxed at a lower rate. We fix (wasteful) government consumption at 19% of GDP in the benchmark economy, see German Statistical Office (2020).

Figure C.1: Labor tax schedule



The government provides two direct transfer programs to support families with children. In 2017, parents received a child benefit of EUR 192 per child and month. Moreover, we let the government pays additional tax financed child support payments to single mothers, which mimic both alimony payments but also subsistence transfers in the real world. We set these monthly child support payments to EUR 576 per child.[29] With average labor earnings of 37,000 Euros, the child transfer function consequently reads

$$t(k,i) = \begin{cases} 0.1245 \times \bar{y} & \text{if } k \in \{1,2\} \text{ and } i = c \\ 0.4980 \times \bar{y} & \text{if } k \in \{1,2\} \text{ and } i = s \\ 0 & \text{otherwise.} \end{cases}$$

In addition to direct transfers, mothers are compensated for foregone pension contributions while raising children. For each child, they are credited a pension top-up that is equal to the pension top-up of an average earner. In order to account for this in the model, we set the child pension credit to $p_{child}(k = 1) = \bar{y}$ and to zero otherwise, as the two children borne to a mother remain small ($k = 1$) for an average of six years.

[29]In Germany, alimony payments depend on individual income and the age of the child. The so-called "Duesseldorfer Table" specifies the exact amount, see OLG Düsseldorf (2017). For monthly net earnings between 1,500-5,100 Euro, monthly payments amount 360-736 Euro in 2017.

C.7 Parameter Values - Overview

Table C.3 summarizes calibrated parameters as well as their targets. Table C.4 summarizes all model parameters that were taken directly from the literature or estimated directly from data.

Table C.3: Summary of endogenous model parameters

	Value	Target
To target labor supply		Data from Table C.1
Disutility labor ν_f	22.0000	
Disutility labor ν_m	70.0000	
Disutility empl. mean μ_ξ	1.6500	
Disutility empl. var σ_ξ^2	2.5000	
Transition prob. $\pi_h(h_{part} \vert h_{full}, f, \ell < \ell_{full})$	0.9500	
Transition prob. $\pi_h(h_{full} \vert h_{part}, f, \ell < \ell_{full})$	0.1500	
Time costs of children		
- single mothers, $k = 1$, $\zeta_{1,s,f}$	1.5000	
- single mothers, $k = 2$, $\zeta_{2,s,f}$	0.2500	
- couple mothers, $k = 1$, $\zeta_{1,c,f}$	1.2000	
- couple mothers, $k = 2$, $\zeta_{2,c,f}$	0.1875	
- couple fathers, $k = 1$, $\zeta_{1,c,m}$	0.3000	
- couple fathers, $k = 2$, $\zeta_{2,c,m}$	0.0625	
- all paretns, $k = 3$, $\zeta_{0,i,g}$	0.0000	
Others		
Depreciation rate δ	0.0700	Investment/output: 21.0
Technology level Ω	0.9232	Wage per efficiency unit $w = 1$
Discount factor β	0.9785	Closed economy: NFA = 0.00
Consumption tax rate τ_c	0.1617	Government budget balance
Replacement rate κ	0.4133	Pension budget balance

D Datawork

The productivity profiles in this paper are based on administrative data from the German Pension Insurance. In particular we use the 2017 wave of the scientific use-file of the Versichertenkontenstichprobe (FDZ-RV, 2017b) that contains monthly earnings data of 69,520 insured individuals. This is about 0.18% of the actively insured population.[30] We will limit our attention to the male sample population between the ages of 25 and 60, for which we have information on the level of education. We consider observations for the years 2000 to 2016. Our measure

[30] The German pension scheme covered of 38 million actively insured individuals in 2017.

Table C.4: Summary of exogenous or estimated model parameters

	Value (m/f)	Source
Demographics		
Max. age J	99	Assumption
Survival probabilities $\psi^i_{j,g}$	Figure 3	Human Mortality Database (2020)
Gender distribution ϕ_g	0.5078/0.4922	German Microcensus 2017
Share college educated ϕ^g_e	0.3306/0.2776	German Microcensus 2017
Share married $\phi_{c,g}$	0.6754/0.6968	German Microcensus 2017
Share mother (singles) ϕ^s_k	0.4753	German Microcensus 2017
Share mother (married) ϕ^c_k	0.8002	German Microcensus 2017
Retirement age j_r	64	DRV Bund (2019)
Labor market		
Weekly hours full-time ℓ_{full}	0.4030	German Microcensus 2017
Weekly hours part-time ℓ_{part}	0.2100	German Microcensus 2017
Weekly hours minijob ℓ_{mini}	0.1000	Minijob Law 2017
Minijob earnings $\bar{y}_{mini}$	$0.1297\times\bar{y}$	Minijob Law 2017
Others		
Child tranfers $t(k \in \{1,2\}, s)$	$0.4980\times\bar{y}$	OLG Düsseldorf (2017)
Child tranfers $t(k \in \{1,2\}, c)$	$0.1245\times\bar{y}$	German Statistical Office (2020)
Pension top-up $p_{child}(1)$	$\bar{y}$	DRV Bund (2020)
Returns to scale $\upsilon(j,k,i)$		New OECD equiv. scale
Pension contribution rate τ_p	0.1870	DRV Bund (2020)
Pension share minijobs ϱ	0.8021	DRV Bund (2020)
International interest rate $\bar{r}$	0.0300	
Capital share in production α	0.3000	
Intert. elast. of substitution σ	2.0000	Heathcote et al. (2014)
Frisch elast. of labor supply χ_g	0.4000/0.7500	Keane (2011)

of monthly labor earnings comprises income from regular work and short-term unemployment (up to one year).We consider short-term unemployment as an employment type, since individuals are productive when searching for a new job. We sum up monthly earnings observations to construct an annual earnings measure for each individual.

We use the same dataset, data cleaning process and estimation strategy as outlined in detail in Kindermann and Pueschel (2023). However, our approach differs with respect to low earnings individuals. Kindermann and Pueschel (2023) define a low-earnings group that is excluded from the sample. We, instead, simply exclude minijob worker from the sample, as we also treat them separately in the model. Hence, we use the empirical earnings profiles to calibrate the productivity processes for full time- and part-time worker in our model.

D.1 Statistical Model

We describe the earnings dynamics of the sample by a standard AR(1) process in logs. We therefore split the sample according to an individuals' education level $e \in \{0, 1\}$. $e = 0$ summarizes all individuals with high school education, while $e = 1$ indicates the college educated workforce. For each education group, we estimate the statistical model

$$\log(y_{iejt}) = \kappa_{t,e} + \theta_{j,e} + \eta_{iejt} \quad \text{with} \quad \eta_{iejt} = \rho_e \eta_{iej-1,t-1} + \varepsilon_{iejt}, \tag{29}$$

for labor earnings y_{iejt} of an individual i with education e at age j in year t. $\kappa_{t,e}$ is a year fixed effect that controls for earnings changes along the business cycle. $\theta_{j,e}$ is an age fixed effect that informs us about the age-earnings relationship. The noise term ε_{iejt} is assumed to follow a normal distribution with mean 0. Furthermore, we let the stochastic process start from its long-run variance σ_e^2. This means that

$$\varepsilon_{iejt} \sim N(0, \sigma_{\varepsilon,e}^2) \quad \text{and} \quad \eta_{ie20t} \sim N\left(0, \sigma_e^2\right) \text{ with } \sigma_e^2 = \frac{\sigma_{\varepsilon,e}^2}{1 - \rho_e^2}.$$

We use a generalized method of moments estimator to determine the parameters of this model. We thereby control for the fact that the data are top-coded.

D.2 Moment Conditions and Estimation

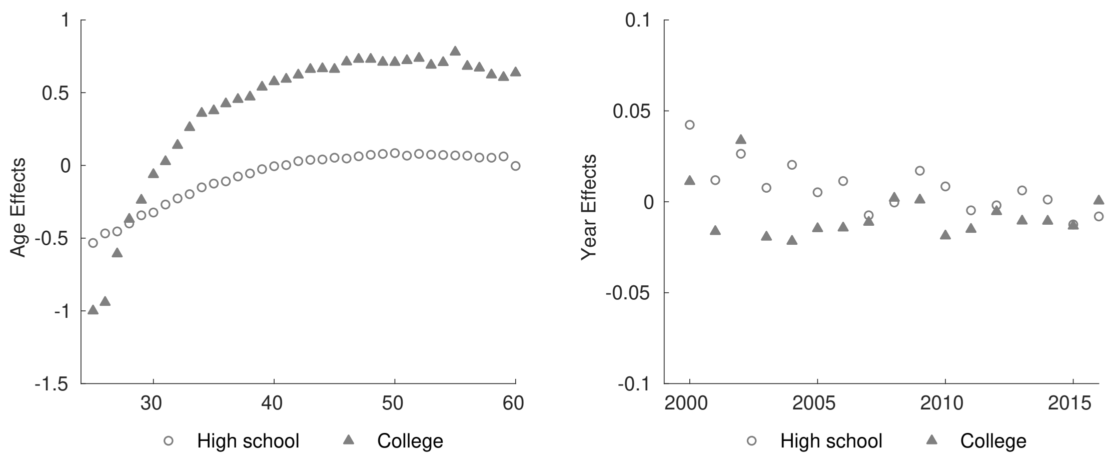
To estimate the statistical model in (29) with our data, we have to determine a total of 110 parameters:

1. 34 year fixed effects $\kappa_{t,e}$ for the years 2000 to 2016 and the education levels $e \in \{0, 1\}$;
2. 72 age fixed effects $\theta_{j,e}$ for the ages 25 to 60 for each education level e;
3. the two unconditional variances σ_e^2;
4. the two autocorrelation parameters ρ_e.

In order to estimate these parameters, we use the labor earnings data y_{iejt}^p to calculate the empirical moments that correspond to the means E_{ejt}, censoring shares P_{ejt}, variances Var_{ejt} and covariances Cov_{ejt} for each education level e, age j and year t. We use these empirical moments to calculate a residual sum of squares measure. We use a diagonal weighting matrix that has the inverse of the squared standard errors of the empirical moments on the diagonal. To minimize the residual sum of squares and account for multiple local minima, we use the method of simulated annealing, see Du and Swamy (2016). We estimate parameters separately for each education level e.

The results of this estimation process are quite standard in the sense that the estimates exhibit typical life cycle labor earnings profiles, a significant college wage premium as well as a high auto-correlation of earnings, see left panel of Figure D.1. We will use these estimates as prime inputs into the calibration of our quantitative model. Yet, as the statistical model describes labor earnings and not labor productivity, we can not use the estimated parameters as direct inputs. The left panel of Figure D.1 visualizes the point estimates of the age fixed effects by education level. Up to the age of 45, earnings steeply increase

Figure D.1: Age fixed-effects and year fixed-effects



for both education groups, especially so for the college educated. Afterwards, they stagnate or decline slightly for the rest of an individual's working life. This shape of life cycle earnings is quite common in the empirical literature and has been found for other countries as well, see for example Heckman et al. (1998) or Casanova (2013). The college-wage premium implied by these profiles is equal to 60 percent, which is in line with empirical findings (OECD, 2016). The right panel of the figure shows the year fixed effects. These are generally small relative to the age effects and exhibit some cyclical dynamics. Table C.2 summarizes the estimation results for the residual earnings process.

E Further Simulation Results

Figure E.1: Changes in employment rates men